MATH–133001

This question paper consists of 3 printed pages, each of which is identified by the reference MATH–1330

Only approved basic scientific calculators may be used.



Examination for the Module MATH–1330

(May/June 2000)

**Financial Mathematics**

Time allowed : 2 hours

Answer **four** questions.
All questions carry equal marks.

**1. (a)** Find the complete solution set of the system of equations

$$\begin{aligned} x + y + 3z - u + v &= 0 \\ 2x + 2y + 7z - 3u + v &= 0 \\ -x - y - 4z + u - 2v &= 0 \\ 3x + 3y + 10z - 4u + 2v &= 0. \end{aligned}$$

Write down a spanning set for the solution set.

**(b)** A manufacturer produces two kinds of tennis rackets: regular and deluxe. Regular rackets require two hours for making the frame, one hour for stringing and two hours for finishing, while the deluxe models require one, three and two hours respectively for these operations. The profit is £8 on regular and £10 on deluxe rackets. The company has available 120 hours for making frames, 150 hours for stringing and 140 hours for finishing. Find the vertices of the feasible set, and hence find how many of each kind of racket the company should produce to maximise its profit.

**2. (a)** (*i*) Find the values of $k$ for which the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 7 & k \end{bmatrix}$ has an inverse.

(*ii*) Calculate the inverse of this matrix for the case $k = 10$.

(*iii*) Hence, or otherwise, solve the system of equations

$$\begin{aligned} x + 2y + 3z &= 13 \\ 2x + 5y + 6z &= 32 \\ 3x + 7y + 10z &= 47. \end{aligned}$$

(*iv*) If there errors of up to 0.25 in the values of the constants on the right hand sides of the above equations, what are the maximum errors in the computed solutions for $x$, $y$ and $z$?

**(b)** Say what is meant by a *linearly independent* set of vectors.

For each of the following sets of vectors in $\mathbb{R}^4$, say whether or not it is linearly independent, giving reasons.

(*i*) $\{(1,2,0,0),\ (0,1,2,0),\ (0,0,1,2),\ (2,0,0,1)\}$

(*ii*) $\{(1,2,0,0),\ (0,1,2,0),\ (0,0,0,0)\}$

(*iii*) $\{(1,2,0,0),\ (0,1,2,0),\ (1,3,2,0)\}$

(*iv*) $\{(1,2,0,0),\ (0,1,2,0),\ (0,0,1,2)\}$

**3.** **(a)** Let $A$ be the matrix $\begin{bmatrix} 4 & 1 & 3 \\ 1 & 4 & 3 \\ 1 & 1 & 6 \end{bmatrix}$.

(*i*) Find the eigenvalues and the corresponding eigenvectors of the matrix $A$.

(*ii*) Explain whether or not $A$ can be diagonalised. *If* $A$ can be diagonalised, write down a matrix $P$ and a diagonal matrix $D$ such that $P^{-1}AP = D$.

(*iii*) What are the eigenvalues and eigenvectors of the matrix $A^3$?

**(b)** Find the value of $x$ which satisfies the equation $\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 6 & 8 & 10 \\ 3 & 6 & 10 & 12 & 15 \\ 4 & 8 & 12 & 17 & 21 \\ 7 & 8 & 9 & 10 & x \end{vmatrix} = 0.$

**4.** **(a)** A bear eats nuts, berries and grubs. The following table gives the nutritional values of these foods in grammes per handful.

| | nuts | berries | grubs |
|---|---|---|---|
| protein | 140 | 90 | 160 |
| fat | 200 | 20 | 100 |
| fibre | 100 | 4 | 70 |
| minutes required to gather a handful | 15 | 10 | 20 |

The bear requires 600 grammes of protein per day, at least 300 of which must come from eating grubs. In addition, the bear requires 400 grammes of fat and 800 grammes of fibre, but more fibre must be from nuts and berries than from grubs. Use the following steps to set up a procedure for deciding the combination of foods that will meet the bear's dietary requirements and minimise the time spent gathering food.

(*i*) Let $x$, $y$ and $z$ be the numbers of handfuls of nuts, berries and grubs that the bear needs to find. Write down the system of inequalities involved, and the quantity to be minimised.

(*ii*) Set up the simplex tableau for this problem.

(*iii*) Carry out ONE pivoting operation on this tableau, explaining carefully how you chose the pivot element. [**Do not** carry out any subsequent pivoting operations.]

**(b)** Find the stable matrix for the Markov process whose transition matrix is

$$\begin{bmatrix} 1 & 0 & 1/4 & 1/6 \\ 0 & 1 & 1/6 & 0 \\ 0 & 0 & 1/4 & 1/2 \\ 0 & 0 & 1/3 & 1/3 \end{bmatrix}.$$

**5. (a)** An investor is considering purchasing shares in one of three companies: Wessex Water (WW, the safe option), Midlands Carparts (MC, speculative) or Flaky Dotcom (FD, high risk). If economic growth in the coming year is strong then the WW stock should increase in value by £3000, MC by £6000 and FD by £15,000. If growth is average then the WW and MC holdings should each go up by £2000 and the FD holding by £1000. If growth is weak then the WW shares should increase by £1000 but the MC and FD shares decrease by £3000 and £10,000 respectively.

Write down a $3 \times 3$ payoff matrix showing the investment gains for the possible share purchases and levels of economic growth.

**(b)** In a two-person game, explain what is meant by $(i)$ a *pure strategy*, $(ii)$ a *mixed strategy*, for the player R.

**(c)** Determine an optimal mixed strategy for R for the game with payoff matrix

$$\begin{bmatrix} 1 & 5 & 4 \\ 4 & 0 & 1 \end{bmatrix}.$$

Write down optimal strategies for R for the games with payoff matrices

$$(i) \quad \begin{bmatrix} -2 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}, \qquad (ii) \quad \begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & -2 \end{bmatrix}.$$

**END**