This question paper consists of 2 printed pages,
each of which is identified by the reference MATH-1225.
Only approved basic scientific calculators may be used.

Examination for the module MATH-1225
(January 2007)

INTRODUCTION TO GEOMETRY

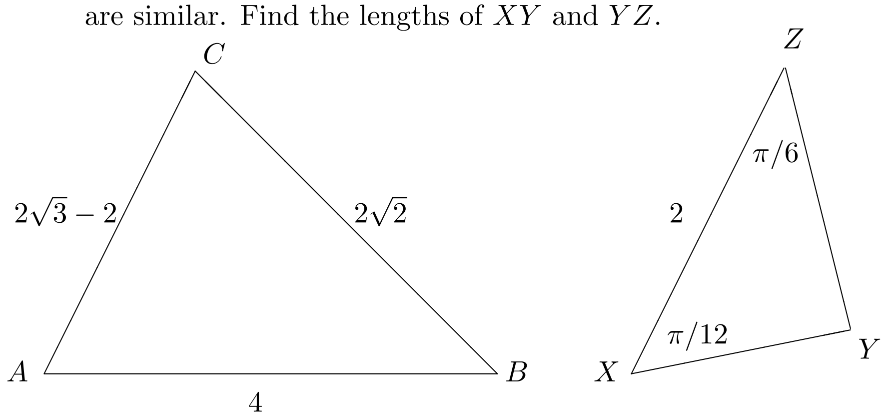
Time allowed : 2 hours

Answer **four** questions. All questions carry equal marks.

1. (a) (i) State Pythagoras' Theorem as given in the lectures.
   (ii) State its converse.
   (iii) In each of the cases below, the three sides of a triangle are given. Determine whether or not the triangle is right-angled, stating whether you used Pythagoras' Theorem or its converse.
   (1) 5.6, 6.5, 3.3. (2) 7.6, 8.3, 3.6.

   (b) State and prove the Sine Rule.

   (c) Show that the triangles ABC and XYZ below (which are not drawn to scale) are similar. Find the lengths of XY and YZ.



2. (a) The point (x, y) moves so that its distance from the line $x - 2y = 1$ is equal to its distance from the point $F = (1, 3)$. State the geometric shape of the locus of the point, and find its equation, stating clearly any results you use.

   (b) Sketch the hyperbola given by the equation $x^2 - 2x - 4y^2 - 24y = 51$. Find its centre, asymptotes, eccentricity and foci.

Continued ...

3. (a) The point P on the parabola $y^2 = 4ax$ is given parametrically by $P = (ap^2, 2ap)$ for $a > 0$.
   (i) Write down the equation for the tangent line at P.
   (ii) Suppose that $Q = (27, -18)$ lies on the parabola. Find a.
   (iii) Suppose that R lies on the lower half of the parabola and its tangent line goes through $(-7, \frac{5}{2})$. Find R.
   (iv) Find the point where the tangents at Q and R intersect.

   (b) The curve called the Witch of Agnesi is given by the equation
$$y(x^2 + b^2) = b^3$$
   where $b > 0$ is a constant. It can be given parametrically by
$$x = bt, \qquad y = \frac{b}{1+t^2}.$$
   Show that these define the same set of points.

   (c) Sketch the curve in the case that $b = 1$.

4. (a) Define polar coordinates and show how they can be calculated from Cartesian coordinates.

   (b) Consider the Cartesian coordinates (x, y). Show that when the axes are rotated through an angle α that the new coordinates (X, Y) are given in terms of the old coordinates (x, y) by
$$\begin{aligned} X &= x\cos\alpha + y\sin\alpha, \\ Y &= -x\sin\alpha + y\cos\alpha. \end{aligned}$$

   (c) Use the above to show that the equation $39x^2 + 70\sqrt{3}xy + 109y^2 = 64$ represents an ellipse and sketch it.

5. (a) (i) The plane P given by $ax + by + cz = d$ goes through the points $(1, -3, 4)$, $(0, 5, -2)$, and $(1, 7, 2)$. Determine a, b, c and d. Hence write down any normal to this plane.
   (ii) Consider the plane Q given by $5x + 11y - 3z = 7$. To the nearest degree, find the angle between the planes P and Q.

   (b) State Euler's relation connecting the number of edges, faces and vertices of a polyhedron. Describe *one* of the five Platonic solids and show that it satisfies Euler's equation.

   (c) A *truncated octahedron* is a polyhedron with 14 faces, of which 6 are squares and 8 hexagons. How many edges does a truncated octahedron have? How many vertices?

The End