MATH–037001

This question paper consists of 2 printed pages, each of which is identified by the reference MATH–037001

Only approved basic scientific calculators may be used.

Examination for the Module MATH–0370

(May 2008)

Introduction to Applied Mathematics 2

Time allowed: 2 hours

Attempt **all** questions. Marks for each question are given in the right-hand margin. There are 80 marks available.

Take $g = 10\,\mathrm{m/s^2}$ unless stated otherwise. State clearly any formulas that you use.

1. A particle moves such that its position $\underline{r}(t)$ is

$$\underline{r}(t) = (t^2 - 2t)\,\underline{i} + (2t^3 - 9t^2 + 12t)\,\underline{j} + (2t^3 - 3t^2)\,\underline{k}.$$

(a) Find the particle's velocity and acceleration.
(b) Is there a time at which the particle is at rest? [6 marks]

2. A block of mass 0.5 kg is subject to a force

$$\underline{F}(t) = \cos t\,\underline{i} - \sin t\,\underline{j}.$$

At time $t = 0$, the particle is at the origin and is moving with velocity $2\,\underline{k}$. Find the position of the particle $\underline{r}(t)$. [6 marks]

3. (a) A stone is thrown upwards with speed 11 m/s from the top of a cliff that is 12 m high. Calculate when the stone reaches the bottom of the cliff, and the speed it is travelling when it does so.
(b) Another stone is thrown from the top of the cliff, one second (1 s) after the first. At what speed and in what direction should it be thrown in order to reach the bottom of the cliff at the same time as the first? How fast is it travelling when it reaches the bottom? [10 marks]

4. A girl stands at the top of a mound that is 2.4 m high. The mound slopes downwards with an angle of 30°. She throws a stone with an initial speed of 6 m/s at angle 60° above the horizontal. Show that the stone lands exactly at the bottom of the slope. [8 marks]

continued ...

5. (a) A golfer hits a golf-ball over level ground. The ball has an initial speed of 45 m/s, and it starts at an angle of 45° above the horizontal. What is the maximum height of the ball, and how far away from the golfer does it land?
(b) The golfer is trying to hit the ball so that it lands in a small hole that is 175.5 m away. At what angle should she hit the ball in order for the ball to land in the hole, if she hits it with the same initial speed? [8 marks]

6. A rope is slung over a smooth tree branch. Two children hang on to either end of the rope. The heavier child, who has a mass of 42 kg, is found to descend with an acceleration of $2\,\mathrm{m/s^2}$. Find the tension in the rope and the mass of the lighter child. [6 marks]

7. A stone of mass 2 kg hangs from a rope that is attached to the ceiling of a lift. The lift starts at the ground floor, accelerates upwards at $2\,\mathrm{m/s^2}$ for 3 s, travels at a constant speed for 10 s, and then decelerates at $3\,\mathrm{m/s^2}$. Draw diagrams showing all the forces on the stone at each stage of the motion. [6 marks]

8. A block of mass 10 kg is pushed up a rough slope that is inclined at 53.13° above the horizontal. A horizontal force of 600 N is applied to the block. The coefficient of friction is 0.5. Draw a diagram showing the forces on the block, and calculate its acceleration. [8 marks]

9. A stone A of mass 2 kg is moving with speed 3 m/s to the right. It collides with a second stone B, which has mass 3 kg and which is moving at speed 1 m/s in the opposite direction. If the coefficient of restitution is $e = 0.25$, find the speeds and directions of motion of the stones immediately after impact. [8 marks]

10. A ball of mass 2 kg is dropped onto the horizontal ground from a height of 10 m. The coefficient of restitution is $e = 0.8$. Calculate
(a) the total mechanical energy at the instant the ball is released;
(b) the speed with which the ball hits the ground;
(c) the total mechanical energy after the bounce;
(d) the height to which the ball rises after the bounce. [8 marks]

11. The coefficient of friction between the road and the tyres of a car is 0.25. The mass of the car is 1000 kg. What is the maximum speed at which the car can be driven without slipping around a corner of radius 160 m, if the road is horizontal? Draw a diagram showing all the forces acting on the car if it is driven at this maximum speed. [6 marks]

END