MATH–037001

© UNIVERSITY OF LEEDS

Examination for the Module MATH–0370

(May/June 2007)

Introduction to Applied Mathematics 2

Time allowed: 2 hours

Attempt **all** questions. Marks for each question are given in the right-hand margin. There are 80 marks available.

Take $g = 10 \, \text{m/s}^2$ unless stated otherwise. State clearly any formula you use, and your reason for using it.

1. A particle moves such that its position $\underline{r}(t)$ is
$$\underline{r}(t) = 2\cos t \, \underline{i} + 3t \, \underline{j} - t^4 \, \underline{k}.$$

   Find the particle's velocity, speed and acceleration. Is there a time at which the particle is at rest? [8 marks]

2. A block of mass $2 \, \text{kg}$ is subject to a force
$$\underline{F}(t) = 4\cos 2t \, \underline{i} + 6 \, \underline{k}.$$

   At time $t = 0$, the particle is at the origin and is moving with speed $2\underline{j} - 4\underline{k}$. Find the position of the particle $\underline{r}(t)$. [8 marks]

3. An escalator is broken and therefore stationary. A person can walk up the stationary escalator in 90 s. Once the escalator is repaired, the person would be carried to the top in 60 s (without walking). How long would it take to reach the top if the person walked up the moving escalator? [6 marks]

4. The brakes on a bus are able to decelerate the bus at a rate of $4 \, \text{m/s}^2$. The bus is approaching a bus-stop at $20 \, \text{m/s}$.
   (a) How far away from the bus-stop must the driver apply the brakes?
   (b) If the driver applies the brakes 1 s late, by what distance does the bus oveshoot the bus-stop?
   (c) How fast is the bus moving when it passes the bus-stop? [7 marks]

1

continued . . .

5. A locomotive of mass 50 tonnes is pulling two carriages, each of mass 10 tonnes, with an acceleration of $1\,\text{m/s}^2$. Each carriage is subject to $2\,\text{kN}$ of friction. Draw a diagram that clearly indicates all the forces acting on the locomotive, the first carriage and the second carriage. Explain carefully how you calculate the values of all the forces. [8 marks]

6. A stone is launched in a catapult from a cliff top $100\,\text{m}$ above the sea. The initial speed of the stone is $50\,\text{m/s}$. Suppose that it is launched with an angle $\alpha = 30°$ above the horizontal. Let x be the horizontal distance travelled, and let y be the vertical distance above the sea.
(a) Calculate $x(t)$ and $y(t)$ (stating clearly the origin of your coordinate system), and find a relationship between x and y that does not involve the time t.
(b) Calculate the distance D from the base of the cliff where the stone falls into the sea. [9 marks]

7. A block of mass m is on a table and is connected to a $2\,\text{kg}$ weight hanging over the edge of a table by a light inextensible string. The block slides across the rough table with acceleration $2\,\text{m/s}^2$, and the coefficient of friction between the block and the table is $\mu = 0.2$. Calculate m and find the tension in the string. [7 marks]

8. A block of mass $20\,\text{kg}$ is sliding down a rough slope at a constant speed. The slope is inclined at an angle of $30°$ to the horizontal. Find all forces acting on the block, and the coefficient of friction. [7 marks]

9. A stone A of mass $2\,\text{kg}$ is sliding on a smooth surface at $6\,\text{m/s}$. It strikes a stationary stone B of mass $4\,\text{kg}$. The coefficient of restitution of the collision is $e = 0.5$. Calculate the speeds and directions of motion of the two stones after the collision. [7 marks]

10. On the planet Zorg, the value of the gravitational acceleration is $g = 16\,\text{m/s}^2$. A ball of mass $0.5\,\text{kg}$ is dropped onto the horizontal ground from a height of $2\,\text{m}$. The coefficient of restitution is $e = 0.8$. Calculate
(a) the total mechanical energy at the instant the ball is released;
(b) the speed with which the ball strikes the ground;
(c) the total mechanical energy after the bounce;
(d) the height to which the ball rises after the bounce. [8 marks]

11. The coefficient of friction between the road and the tyres of a motorbike is 0.2. At what speed is it safe to drive round a corner of radius $50\,\text{m}$ if the road is horizontal? [5 marks]

END