MATH-013101

Only approved basic scientific calculators may be used.

This question paper consists of 3 printed pages, each of which is identified by the reference MATH-0131, together with 2 pages of formula sheet.

Examination for the Module MATH-0131

(January 2003)

Elementary Differential Calculus: Version 3

Time allowed : 2 hours

Attempt all the questions in Section A and **three** questions from Section B.

Each question in Section A carries 2 marks, and each question in Section B carries 20 marks. You must show your working in answers to all questions. A formula sheet is supplied with this paper.

SECTION A

Attempt all the questions in Section A.

A1. Expand $(2x-5)^2$.

A2. Evaluate $27^{-1/3}$.

A3. Evaluate $\dfrac{2x^3y}{y^2z^4}$ when $x=4$ and $y=3$ and $z=2$.

A4. What is the value of $\log_2 \frac{1}{4}$?

A5. Factorize $4x^2-9$.

A6. Solve the equation $x^2-4x=5$.

A7. Find the equation of the straight line with gradient 2 passing through the point $(1,6)$.

A8. What is the gradient of a line perpendicular to the line $y=8x-2$?

A9. The angle θ is acute and $\sin\theta=\frac{1}{4}$. Find the values of $\cos\theta$ and $\tan\theta$, leaving your answers as exact expressions involving square roots.

A10. In the triangle ABC, the angle at A is 30° and the angle at B is 90°. If BC has length 4cm, what is the length of AC?

A11. Without using a calculator, find exact expressions for $\sin\frac{\pi}{6}$ and $\cos\frac{\pi}{6}$.

A12. Find all the exact values of θ between 0 and 2π radians which satisfy the equation $\cos\theta = \frac{1}{\sqrt{2}}$.

A13. Find $\frac{dy}{dx}$ when $y = 3x^5$.

A14. Find $\frac{dy}{dx}$ when $y = x^{1/2} + \frac{1}{x^{1/2}}$.

A15. Find $\frac{dy}{dx}$ when $y = (2x+1)^{12}$.

A16. Find $\frac{dy}{dx}$ when $y = 2\cos x - 7\sin x$.

A17. Find $\frac{dy}{dx}$ when $y = e^{\tan x}$.

A18. Find $\frac{dy}{dx}$ when $y = \ln(2x^2+1)$.

A19. Find $\frac{d^2y}{dx^2}$ when $y = 2x^4 - 9x$.

A20. Find the equation of the circle with centre $(2, 3)$ and radius 3.

SECTION B

Attempt **three** questions from Section B.

B1. **(a)** Find the turning points of the curve $y = 2x^3 - 3x^2 + 1$, and classify them as local maxima or minima. Give a rough sketch of the curve. *You must show all your working.*

Question B1 continues ...

(b) Find the gradient of the curve $y = x^3 + 2\ln x$ at the point where $x = 1$.

Find the equation of the tangent to the curve at this point.

B2. **(a)** The point P has coordinates $(1, 4)$. Find

(i) the equation of the straight line throught P with gradient 5;

(ii) the coordinates of the point of intersection, Q, of this line with the line whose equation is $x + y = 11$;

(iii) the distance PQ;

(iv) the equation of the line perpendicular to PQ that passes through the origin O.

(b) A circle has centre C with coordinates $(3, 0)$, and passes through the point A with coordinates $(0, 3)$. Find

(i) the radius of the circle;

(ii) the equation of the circle;

(iii) the gradient of the line AC;

(iv) the equation of the tangent to the circle at A.

B3. **(a)** Sketch the graph of the function $\sin x$ in the range $0 \leq x \leq 2\pi$ (radians). For the same range of values of x sketch the graphs of $y = \sin 2x$ and $y = \sin \frac{x}{2}$. Label in radians the points at which the graphs cross the x-axis.

(b) Use the identity $\sin^2 x + \cos^2 x = 1$ to find all solutions of the equation
$5\sin^2 x + \cos^2 x = 2$ in the range $0 \leq x \leq 2\pi$.

(c) The angle θ is acute and $\cos\theta = \frac{2}{5}$. Find (without a calculator) exact values of $\sin\theta$, $\tan\theta$, $\sin 2\theta$ and $\cos 2\theta$.

B4. Differentiate each of the following functions with respect to x.

(a) $y = \sqrt{x^6 - 2x}$;

(b) $y = e^{3x}\cos 2x$;

(c) $y = x\ln x - x$;

(d) $y = \dfrac{x - 7}{x^2 + 1}$;

(e) $y = \arctan(x^3)$.

END