MATH–013101

This question paper consists of 3 printed pages, each of which is identified by the reference MATH–013101

Only approved basic scientific calculators may be used.

Examination for the Module MATH–0131

(January 2007)

Elementary Differential Calculus (Version 3)

Time allowed: 2 hours

Attempt **all** questions in Section A and any *three* questions from Section B.

Each question in Section A carries 2 marks, each question in section B carries 20 marks.
You must show your working in answer to all questions.
A formula sheet is supplied with this paper.

SECTION A

Attempt **all** the questions in Section A

A1. Expand $(x+2)(3-5x)$.

A2. Evaluate $16^{-3/4}$.

A3. Evaluate $3x^{3/2}y^2x^{-2}y^{-3}$ when $x = \dfrac{1}{4}$ and $y = 7$.

A4. Find $\log_{27} 9$.

A5. Factorise $x^2 + x - 12$.

A6. Solve the equation $x^2 + 3x - 10 = 0$.

A7. Find the equation of the straight line through the point $(1, 2)$ which is perpendicular to the line $x - 2y + 3 = 0$.

A8. What is the distance between the points $(1, -1)$ and $(-2, 3)$?

A9. The angle θ lies between 0 and $\pi/2$ and $\sin\theta = \dfrac{1}{3}$. Find $\cos\theta$ and $\tan\theta$ leaving your answers as exact expressions involving square roots.

A10. Find the equation of the circle with centre $(-1, 1)$ and radius 2.

A11. Find $\dfrac{dy}{dx}$ when $y = x^{3/4}$.

A12. Find $\dfrac{dy}{dx}$ when $y = 2x^3 + x^2 + 8$.

A13. Find $\dfrac{dy}{dx}$ when $y = \sqrt[3]{x^2 + 3}$.

A14. Find $\dfrac{dy}{dx}$ when $y = \sin x^3$.

A15. Find $\dfrac{dy}{dx}$ when $y = \dfrac{x^3 + x}{7x + 2}$.

A16. Find $\dfrac{dy}{dx}$ when $y = e^{3x} \cos x^2$.

A17. Find $\dfrac{dy}{dx}$ when $y = \ln(\tan x + 6)$.

A18. Find $\dfrac{d^2y}{dx^2}$ when $y = 5x^2 + 7x^3$.

A19. Find the tangent to the curve $y = x^2 + 2x - 3$ at the point $(-1, -4)$.

A20. Without using a calculator, find an exact expression for $\sin(5\pi/6)$.

SECTION B

Attempt **three** questions in Section B

B1. **(a)** Sketch the graph of $y = \cos\theta$, for θ in the range $-2\pi \leq \theta \leq 2\pi$ labelling the values of θ where the graph crosses the horizontal axis and where y has minimum and maximal values.

(b) Find all values of θ (in radians) between -2π and 2π, such that $\cos\theta = \dfrac{\sqrt{3}}{2}$.

(c) Using the formula for $\cos(A + B)$ from the formula sheet, show that $\cos(\theta - \frac{\pi}{2}) = \sin\theta$. Use the result from the previous part of the question to find all values of θ between $-\pi$ and π such that $\sin\theta = \dfrac{\sqrt{3}}{2}$.

B2. **(a)** The points A and B have coordinates $(2, 3)$ and $(-1, 4)$ correspondingly. Find:

(i) the equation of the line AB;
(ii) the equation of the line through the origin perpendicular to AB;
(iii) the point where the above two lines meet;

(iv) the distance from the origin to the line AB.

(b) A circle has centre at the point $C = (4, 2)$ and passes through the point $P(1, 6)$. Find:

(i) the radius of the circle;
(ii) the equation of the circle;
(iii) the gradient of the line CP;
(iv) the equation of the tangent to the circle at P.

B3. Differentiate each of the following functions with respect to x.

$$\begin{aligned}
\textbf{(i)} \quad y &= (x^2 - 2x)^3 + \sqrt[3]{x^4 - 1}; \\
\textbf{(ii)} \quad y &= (x^4 + e^x)\sin(2x - 1); \\
\textbf{(iii)} \quad y &= \frac{x^2 + 6}{x(\ln x + 4)}; \\
\textbf{(iv)} \quad y &= \arccos(e^x); \\
\textbf{(v)} \quad y &= (2^x + e)^4.
\end{aligned}$$

B4. **(a)** Find the stationary point of the function given by $y = 2x^3 - 3x^2 - 12x + 6$ and determine whether they are (local) maximum or minimum points.

(b) Find the maximum and minimum values of $7 - 2x + x^2$ for x between 0 and 3.

(c) If y is given as a function of x by $2xy^2 + 3y = 2x + 1$, find $\dfrac{dy}{dx}$ in terms of x and y.

END