L A N C A S T E R   U N I V E R S I T Y

2000 EXAMINATIONS

Part I

PHYSICS - Paper PS1.1

- *Candidates should attempt all those sections identified with the modules for which they are registered.*
- *Candidates who attended PHYS111 &/or PHYS112 attempt sections A &/or B.*
- *Candidates who attended PHYS111a &/or PHYS112a attempt sections C &/or D.*
- *The time allocated is 60 minutes per section.*
- *An indication of mark weighting (30 marks per section) is given by the numbers in square brackets following each part.*
- *In each section attempted, candidates should answer question 1 (10 marks) and* <u>*either*</u> *question 2* <u>*or*</u> *question 3 (20 marks).*
- *Use a separate answer book for each section.*

PHYS110

Section A: Module 111 - Relations, Functions and Series
For candidates who attended PHYS111 (NOT PHYS111a).

A1. (a) The equation of a straight line is

$$3x + 2y - 5 = 0.$$

Find the slope and the intercepts on the x and y axes. Sketch its graph. [3]

(b) Explain briefly what is meant by a *periodic function*. Give one example including a sketch to illustrate your answer. [2]

(c) Use your calculator to find $\log_{10} 5$ and $\log_{10} 8$. Hence find $\log_5 8$. [2]

(d) Given that the exact value of $\tan(\pi/6) = \dfrac{1}{\sqrt{3}}$, find the exact value of $\tan(\pi/12)$. [3]

please turn over

A2. (a) Identify each of the following series as *geometric, arithmetic* or *binomial.* Find the sum of each series.

(i) $\sum_{k=0}^{99}(5+2k)$ (ii) $\sum_{k=0}^{4} 5^k$ (iii) $\sum_{k=0}^{5} {}^5C_{5-k}2^k$

[6]

(b) What is meant by the *convergence* of an infinite series? The series expansion for e^x is

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

Use the D'Alembert ratio test to confirm that the series converges for any finite positive value of x. Use this series to calculate a value for $\sqrt[3]{e}$ to two decimal places. Check your result with a calculator. [8]

(c) The first three terms of the expansion of $(1+x)^r$ for any real number r, are:

$$(1+x)^r = 1 + rx + \frac{r(r-1)x^2}{2!} + ...$$

Use this expansion to show that Einstein's equation $E = mc^2\left(1-\frac{v^2}{c^2}\right)^{-1/2}$ for the total energy E of a particle mass m moving with speed v may be written

$$E \approx mc^2 + \frac{1}{2}mv^2$$

when v is very much less than the speed of light c ($v << c$). [6]

A3. (Calculus methods should *not* be used in this question) A ball is thrown from a point $(0, h)$ so that its x (horizontal) and y (vertical) coordinates are given by:

$$x = ut,$$
$$y = h + vt - \tfrac{1}{2}gt^2.$$

Show that the path of the ball is given by an equation of the form $y = ax^2 + bx + c$. Find a, b and c in terms of u, v, h and g. [6]

Describe briefly the type of curve followed by the ball. Find the coordinates of the point where the ball reaches its maximum height in the case when $h = 7\,\text{m}$, $u = 3\,\text{m s}^{-1}$, $v = 5\,\text{m s}^{-1}$ and $g = 10\,\text{m s}^{-2}$.

Make a sketch of the ball's path including the x-coordinate when it reaches $y = 0$. [10]

If $h = -7\,\text{m}$ and u, v and g are unchanged, under what circumstances will the ball never reach $y = 0$? [4]

Section B: Module 112 - Vectors and Geometry
For candidates who attended PHYS112 (NOT PHYS112a).

B1. (a) Give the name of the conic sections described by the following equations

(i) $x^2 + y^2 = a^2$ [1]

(ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [1]

(ii) $y = ax^2$ [1]

(iii) $y = a/x$ [1]

(b) Give three examples of physical quantities which are scalars. [1]

(c) Give three examples of physical quantities which are vectors. [1]

(d) If $\underline{A} = (1, 2, 3)$ and $\underline{B} = (4, 5, 6)$, compute $\underline{A} \times \underline{B}$. [4]

B2. (a) Show that the line $x + 3y = 1$ is a tangent to the circle $x^2 + y^2 - 3x - 3y + 2 = 0$ and find the coordinates of the point of contact. [7]

(b) What are the coordinates of the centre of the circle and what is the radius of the circle? [7]

(c) Prove, by calculation, that the point (3, 2.5) lies outside the circle. [6]

B3. A body undergoes a displacement $\underline{\boldsymbol{S}} = (1, 2, 2)\,\mathrm{m}$ whilst experiencing a constant force $\underline{\boldsymbol{F}} = (5, -1, 2)\,\mathrm{N}$.

(a) Find the work done. [7]

(b) Compute $|\,\underline{\boldsymbol{F}}\,|$ and $|\,\underline{\boldsymbol{S}}\,|$. [6]

(c) Find the angle θ between $\underline{\boldsymbol{F}}$ and $\underline{\boldsymbol{S}}$. [7]

please turn over

PHYS110a

Section C: Module 111a - Algebra and Functions
For candidates who attended PHYS111a (NOT PHYS111).

C1. (a) Expand the following expressions and write your answers in as simple a form as possible:

(i) $(x+3)(x+4)-x^2$ [2]

(ii) $x(x^2+1)(x-1)+2x$ [2]

(b) What is the gradient of a straight line which passes through the points (1, 3) and (5, 5)? [2]

(c) Solve the following simultaneous equations:

$$\begin{aligned} 3x + y &= 7, \\ x - y &= 1. \end{aligned}$$

[2]

(d) What are the coordinates of the vertex (turning point) of the graph of $y = (x-1)^2$? [2]

C2. (a) Consider the two functions, $y = x^2 - 3x + 2$ and $y = x^2 - 8x + 15$.

(i) Sketch the graphs of the two functions beween $x = 0$ and $x = 6$ on a single pair of axes. What are the intercepts of the two curves with the y-axis? What are the intercepts of the two curves with the x-axis? [8]

(ii) Hence or otherwise, express the two functions in the form $y = (x+\alpha)(x+\beta)$ where α and β are constants. [2]

(iii) For each curve find the coordinates of the vertex (turning point). [4]

(b) A stone is thrown vertically upwards into the air from the ground with an initial speed $u_0 = 20\,\mathrm{ms}^{-1}$. The height of the stone above the ground is given by

$$h = u_0 t + \frac{1}{2} a t^2$$

where $a = -10\,\mathrm{ms}^{-2}$. How long does it take for the stone to hit the ground and what is the maximum height reached by the stone? [6]

C3. (a) The two shorter sides of a right angled triangle have lengths of 6 m and 8 m. Find the length of the third side and find the interior angles of the triangle. [3]

(b) Use the Pythagoras theorem to prove the identity $\sin^2\theta + \cos^2\theta = 1$. [3]

(c) Re-write the following expressions in terms of $\sin\theta$:

(i) $\sin(-\theta)$

(ii) $\sin(\theta + 180°)$

(iii) $\cos(\theta + 90°)$

(iv) $\cos(\theta - 90°)$ [4]

(d) Find all the possible value of θ in the range $\theta° \leq \theta \leq 360°$ which satisfy the equations:

(i) $|\tan\theta| = 1$

(ii) $\sec\theta = \sqrt{2}$ [5]

(e) What are the internal angles of a triangle with:

(i) side lengths of 2 m, 2 m and 3 m,

(ii) side lengths 3 m, 3 m and 4.5 m? [5]

Section D: Module 112a - Functions and Geometry
For candidates who attended PHYS112a (NOT PHYS112).

D1. (a) Without using a calculator, find the values of $\log_{10} 10$ and $\log_{10} 100$. Given that $\log_{10} 5 = 0.699$ find the values of $\log_{10} 50$ and $\log_{10} 500$. [4]

(b) What shape does the equation $x^2 + y^2 = R^2$ represent? What does R tell us? [2]

(c) What is the radius of a circle that encloses the same area as a square of side 2.00 m. [4]

please turn over

D2. Consider the following series of six terms:

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243}.$$

Write down an expression for this series using the $\sum$ notation. Is the series geometric, arithmetic or binomial? Find the sum of the series using the appropriate formula. [6]

Given the infinite series

$$(1+x)^r = 1 + \frac{rx}{1} + \frac{r(r-1)x^2}{2!} + \frac{r(r-1)(r-2)x^3}{3!} + ... \quad \text{for} -1 < x < 1,$$

write down the first three terms of the series for $(1+x)^{-3}$. Hence, find an approximate value for $\frac{1}{1.01^3}$. [8]

Find the infinite series for $\frac{1}{1-x}$ $(-1 < x < 1)$. [6]

D3. (a) The equations for two straight lines are

$$y = 0.5x + 4,$$
$$y = -2x + 5.$$

Make a sketch of the two lines. Find the coordinates of the point where the two lines intersect. Are the two lines perpendicular to each other? Explain your answer. [8]

(b) The equation $y = 4x^2$ represents a parabola. Make a rough sketch of it. Find the coordinates of the points where the straight line $y = 3x + 1$ meets the parabola. [6]

(c) Find the value of c such that the line $y = 3x + c$ is a tangent to the parabola. What are the coordinates of the point where the tangent touches the parabola. [6]