King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/MP36 Medical Imaging and Measurement

Summer 1999

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Plank's constant = 6.6×10^{-34} J s
Gyromagnetic ratio of proton = 2.7×10^{8} rad T^{-1} s^{-1}

SECTION A - Answer SIX parts of this section.

1.1) Define the terms *characteristic acoustic impedance* and *specific acoustic impedance*. Show that particle velocity, u, and acoustic pressure, p, are in phase for a forward travelling, mono-frequency, plane acoustic wave $p(x,t) = A\sin(kx - \omega t)$.

[7 marks]

1.2) Derive an expression for the ultrasound amplitude reflection coefficient across a planar interface between two media of different densities, for plane waves at normal incidence.

[7 marks]

1.3) Describe how fundamental features of electron/positron annihilation are exploited in the design and operation of a positron emission tomography (PET) scanner.

[7 marks]

1.4) Describe the pulse sequence, in appropriate magnetic resonance equipment, that could be used to measure the free induction decay (FID) of the magnetisation of a small tissue volume. How should the measurement scheme be extended if the longitudinal relaxation time of that volume is also to be measured ?

[7 marks]

1.5) Discuss why Tc99m is a suitable radionuclide to use for single-photon emission computed tomography (SPECT) scanning. Does it have any disadvantages for medical imaging?

[7 marks]

1.6) Discuss how the design of an X-ray mammography unit is determined by the physics of the photon/tissue interaction.

[7 marks]

1.7) Calculate the projection (onto any direction in the plane of $\boldsymbol{r}$) of an annular shaped disk,

$$D(\boldsymbol{r}) = c \quad \text{for} \quad a \le |\boldsymbol{r}| \le b$$
$$= 0 \quad \text{otherwise, (with } a, b, \text{ and } c \text{ constants).}$$

[7 marks]

1.8) Demonstrate that the X-ray linear attenuation coefficient is approximately proportional to the density of tissue, when the Compton effect dominates. Clearly state any approximations made.

[7 marks]

SECTION B - answer TWO questions

2) Derive the expression $\Delta f = \frac{2v}{c} f$ for the magnitude of the apparent change in frequency, Δf, of an ultrasound wave of frequency f, when measured by the pulse-echo method from a reflector moving with a velocity v parallel to the direction of propagation of the wave, where c is the velocity of the ultrasound wave. Use a one-dimensional model, and make any assumptions that are appropriate for medical applications

[15 marks]

Describe how the Doppler effect may be exploited in medicine to measure the speed of flowing blood, *in vivo*, with continuous wave ultrasound.

[11 marks]

3) With the aid of labelled diagrams, briefly discuss the basic principles of the operation of a typical single-photon emission computed tomography (SPECT) scanner.

[6 marks]

Develop an approximate attenuation correction procedure to be applied to the projection data for a SPECT image of a bounded region of (radio-) activity enclosed within a larger, non-active object. Be explicit about the approximations made.

[18 marks]

Use simple geometric arguments to indicate how, when using a gamma camera with a pinhole collimator, system magnification and resolution depend on the aperture/source distance.

[6 marks]

4) A proton is placed in a constant magnetic field, $\boldsymbol{B}$, with its magnetic moment, μ, initially at an angle, ϕ, to the field. Show that μ precesses about the direction of $\boldsymbol{B}$ and obtain an expression for the magnitude of the Larmor frequency.
Hint - Use classical arguments and note the following vector identity:

$$(\boldsymbol{a}\times\boldsymbol{b}\times\boldsymbol{c}) = (\boldsymbol{c}\cdot\boldsymbol{a})\boldsymbol{b} - (\boldsymbol{b}\cdot\boldsymbol{c})\boldsymbol{a}$$

[18 marks]

Use the above result to explain why a tissue sample placed in the field $\boldsymbol{B}$ acquires a constant magnetisation, $\boldsymbol{M}$, in the direction of the field, and use qualitative arguments to show how the direction of the magnetisation may be tipped through $\pi/2$ radians.

[8 marks]

What is the difference in energy between the two spin states of ^{1}H, and what are their precessional frequencies, in a magnetic field of 2.4 T?

[4 marks]

5) Explicitly validate the Fourier Slice Theorem for any projection of a two-dimensional object distribution, $D(x,y)$.

[10 marks]

An X-ray system is known to have an isotropic point-spread-function. Show how it is possible to calculate the point-spread-function from measurements on the image of a straight metal wire. Are there any constraints on the dimensions of the wire?

[10 marks]

Obtain an expression for the maximum kinetic energy, E, of the recoil electron in a Compton event occurring in a diagnostic X-ray beam obtained at W kVp? Non-relativistic conditions apply for the recoil electron.

[10 marks]