King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CPMP36 Medical Imaging and Measurement

Summer 2004

Time allowed: THREE Hours

Candidates should answer no more than SIX parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	e_0	=	8.854×10^{-12}	F m^{-1}
Permeability of free space	m_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Speed of light in free space	c	=	2.998×10^{8}	m s^{-1}
Gravitational constant	G	=	6.673×10^{-11}	N m^2 kg^{-2}
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
		=	4.136×10^{-15}	eV s
Boltzmann constant	k_B	=	1.381×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	s	=	5.670×10^{-8}	W m^{-2} K^{-4}
Gas constant	R	=	8.314	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.022×10^{23}	mol^{-1}
Molar volume of ideal gas at STP		=	2.241×10^{-2}	m^3
One standard atmosphere	P_0	=	1.013×10^{5}	N m^{-2}

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SECTION A - Answer SIX parts of this section

1.1) A 5 MHz ultrasound real-time pulse-echo B- mode system is operated with a pulse repetition frequency of 2 kHz. To what depth in soft tissue can images be formed, and what is the maximum frame rate to produce 100-line images? Assume that the speed of sound in soft tissue is 1500 ms^{-1}.

[7 marks]

1.2) Images are obtained using a gamma camera fitted with a pinhole collimator. Derive expressions showing how the linear magnification and geometric efficiency of the system depend on the distance between the aperture and the source. For simplicity, assume that the source is always located along the central axis of the collimator.

[7 marks]

1.3) Briefly discuss the basic principles of the operation of a typical modern PET scanner.

[7 marks]

1.4) Describe what pulse sequence, in an appropriate magnetic resonance setup, could be used to measure the free induction decay of the magnetisation of a small tissue volume. How should the measurement scheme be extended if the longitudinal relaxation time of that volume is also to be measured ?

[7 marks]

1.5) Briefly discuss why Tc-99m is a suitable radionuclide to use for SPECT scanning. Indicate some of its possible disadvantages for medical imaging.

[7 marks]

1.6) Plane waves of ultrasound are incident normally on a planar interface between two media with different characteristic acoustic impedances, Z_1 and Z_2. Show that the ultrasound intensity transmission coefficient at the interface is $\frac{4Z_1 Z_2}{(Z_1 + Z_2)^2}$.

[7 marks]

1.7) Derive the functional form of the projection, onto any direction in the plane of $\boldsymbol{r}$, of the two-dimensional rotationally symmetric distribution

$$D(\boldsymbol{r}) = \exp(-r^2).$$

[7 marks]

1.8) An imaging system has a point-spread function (PSF) expressible as

$$\begin{aligned} P(x,y) &= (\cos x)\cos y \quad &\text{for } |x| \le \pi/2,\ |y| \le \pi/2 \\ &= 0 &\text{otherwise.} \end{aligned}$$

Calculate the modulation transfer function (MTF) for this system.

Note that $\int_{-\pi/2}^{\pi/2} [\exp(ix)] \exp(-2\pi ikx) dx = \frac{2\sin[(1-2\pi k)\pi/2]}{(1-2\pi k)\pi/2}$

[7 marks]

SECTION B - Answer TWO questions

2)a) Describe the principles of a simple "first generation" X-ray system for medical tomographic imaging. List five features of X-radiation, and its propagation in human tissues, that make it suitable for this type of imaging.

[10 marks]

b) Find the lowest energy photon able to impart a kinetic energy W to an electron of mass, m, in a Compton scattering event. Non-relativistic conditions apply for the recoil electron. Deduce that $W \approx 2E^2/mc^2$, where E is the photon energy, and c is the speed of light *in vacuo*.

[10 marks]

c) A narrow, monochromatic, beam of X-rays is incident on a thin layer of material, with density ϱ and consisting of N absorbing atoms per cm^3. The interaction of each atom with the photons is expressed by the total cross section, σ cm^2. Derive expressions for the material's linear attenuation coefficient, mass attenuation coefficient, and half-value layer in terms of N and σ.

[10 marks]

3)a) Contrast the two major types of emission computed tomography systems. Compare their principles of data collection, data input to image, type of radionuclide used, spatial resolution, and accuracy of attenuation correction.

[10 marks]

b) A nuclear medical imaging system utilises an image phantom constructed with the following two-dimensional activity distribution:

$$D(x,y) = A\exp(-|x| - |y|), \quad \text{where } A \text{ is a constant.}$$

Demonstrate the validity of the Fourier Slice Theorem for the data obtained.

[10 marks]

c) A gamma camera system is known to have an isotropic point-spread function. Derive the relationship between the point-spread and line-spread functions, and indicate how the latter could be measured in practice.

[10 marks]

4)a) Define what is meant by the pulse-echo principle used in ultrasound imaging and discuss any approximations implied.

[6 marks]

b) What is the maximum detectable speed of movement in soft tissue, at a depth of 5 cm, with a pulse-wave Doppler device with variable pulse repetition frequency and an ultrasound operating frequency of 5 MHz.
Assume that the speed of sound in soft tissues is 1500 m s^{-1}

[10 marks]

c) Derive the expression $\Delta f = \frac{2v}{C} f$ for the magnitude of the apparent change in frequency, Δf, of an ultrasound wave of frequency f, when measured by the pulse-echo method from a reflector moving with speed v parallel to the direction of propagation of the wave, and where C is the speed of sound. Use a one-dimensional model, stating any assumptions that are appropriate for medical applications.

[14 marks]

5)a) A proton is placed in a constant magnetic field, $\boldsymbol{B}$, with its magnetic moment, $\boldsymbol{\mu}$, initially at an angle, ϕ, to the field. Using only classical arguments:

i) show that $\boldsymbol{\mu}$ precesses about the direction of $\boldsymbol{B}$.

[8 marks]

ii) obtain an expression for the magnitude of the Larmor frequency.
Note the following vector identity $(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} = \boldsymbol{b}(\boldsymbol{c} \bullet \boldsymbol{a}) - \boldsymbol{a}(\boldsymbol{b} \bullet \boldsymbol{c})$.

[10 marks]

b) The resonance frequency for a proton in a 7 tesla field is 300 MHz. Obtain a value for the gyromagnetic ratio, γ, and find the energy difference between the two energy states of a proton in a 1 tesla magnetic field.

[6 marks]

c) A sample of liver tissue with a longitudinal relaxation time constant T_1 = 200 ms and a transverse relaxation time constant T_2 = 50 ms is placed in a constant magnetic field, pointing along the z-direction. What will the z-component of the bulk magnetisation be 100 ms after a radio-frequency pulse has flipped the bulk magnetisation vector into the transverse plane? What will the transverse component be? What will the magnitude of the magnetisation be?

[6 marks]