SECTION A - Answer SIX parts of this section.

1.1) Calculate the projection of a two-dimensional, planar, square object with constant 'density', C, onto the direction defined by its diagonal.

[7 marks]

1.2) Describe the essentials of a "first generation" X-ray system for medical computerised tomography. List those features of X-radiation, and its propagation in human tissues, that make it suitable for this type of imaging.

[7 marks]

1.3) Show how the intensity of a plane ultrasound pressure wave of form $p(x,t) = A\sin(kx - wt)$ depends on its amplitude and the characteristic acoustic impedance.

[7 marks]

1.4) It is desired to obtain an ultrasound pulse-echo B-mode image to a depth of 20 cm in the body. What is the maximum pulse repetition frequency at which the system can be operated? What image frame rate is possible if 120 image lines per frame are required? (The speed of sound in soft tissues is 1500 m s^{-1}.)

[7 marks]

1.5) Determine the energy difference between the low and high energy states of a proton in a 1 tesla magnetic field, given that the resonance frequency for a proton in a 7 tesla field is 300 MHz.

[7 marks]

1.6) Images are obtained using a gamma camera fitted with a pinhole collimator. Use simple geometric arguments to indicate how the magnification and resolution of the system depend on the distance between the aperture and the source.

[7 marks]

1.7) Sketch and name three types of collimator, other than the pinhole collimator, used with gamma cameras and indicate how the image in the crystal relates to the object.

[7 marks]

1.8) An imaging system is found to have a point spread function of the form $A\exp(-|x| - |y|)$, with A constant. Obtain an expression for the phase transfer function of this system.

[7 marks]

SECTION B - answer TWO questions

2) (a) A narrow, monochromatic beam of X-rays is incident on a thin layer of material containing N atoms per cm^3. The interaction of each atom with the photons is expressed by the total cross section, s cm^2. Derive expressions in terms of N and s for (i) the linear attenuation coefficient of the material, (ii) its half-value layer and (iii) its mass attenuation coefficient. State the dimensions of the calculated quantities.

[8 marks]

(b) Obtain an expression for the energy of the *softest* photon able to impart a kinetic energy, W, to a recoil electron in a Compton scattering event. Non-relativistic conditions apply for the recoil electron.

[10 marks]

(c) An X-ray system is known to have an isotropic point-spread function (PSF). Derive the relationship between the point-spread and line-spread functions, making clear any assumptions made. Indicate how it would be possible in practice to obtain the modulation transfer function (MTF) from the image of a straight wire. What constraints are there on the size and material of the wire?

[12 marks]

3) (a) Describe the analytic method used for recovering the frequency shift of the echoes in a 'spectral Doppler' pulse-wave device.

[10 marks]

(b) What is the maximum speed of movement detectable in soft tissue, at a depth of 5 cm, with a pulse-wave Doppler device with variable pulse repetition frequency (PRF) and an ultrasound operating frequency of 5 MHz? (The speed of sound in soft tissues is 1500 $m\,s^{-1}$.)

[12 marks]

(c) Derive an expression for the amplitude reflection coefficient of an ultrasound wave passing from muscle into liver, at normal incidence, across a planar interface located at $x=0$. The two tissues have different mass densities and different values for the speed of sound.

[8 marks]

4) (a) Contrast positron emission tomography (PET) and single photon emission computed tomography (SPECT) with respect to principle of data collection, measured input to image recovery, type of radionuclide used, spatial resolution, and accuracy of attenuation correction.

[10 marks]

(b) Explain how the measured data may be corrected for photon attenuation in a PET scanner. Calculate, explicitly, the form of the attenuation correction for a patient whose body cross-section may be assumed to be circular, with constant linear attenuation coefficient.

[12 marks]

(c) Discuss briefly the advantages and disadvantages of using the radionuclide Tc99m for single photon emission computed tomography (SPECT) scanning.

[8 marks]

5) (a) A proton is placed in a constant magnetic field, *B*, with its magnetic moment, *m*, oriented at an angle *q* to the field direction. Show that *m* precesses about the direction of the field.

[8 marks]

(b) Describe the pulse sequence that, with appropriate magnetic resonance equipment, could be used to measure the free induction decay (FID) of the magnetisation of a small tissue volume. How should the measurement scheme be extended if the longitudinal relaxation time of that volume is also to be measured ?

[8 marks]

(c) ^{1}H nuclei are exposed a field of strength *B* tesla at a temperature T ^{0}C. Derive expressions for the fractions of nuclei that are in the upper and lower energy states.

[10 marks]

(d) A sample of liver tissue has a longitudinal relaxation time constant T_1 = 500 ms and a transverse relaxation time constant T_2 = 40 ms is placed in a constant field of strength *B* tesla, pointing along the *z*-direction. Determine the z-component of the bulk magnetisation 500 ms after a radio-frequency pulse has flipped the bulk magnetisation vector into the transverse plane.

[4 marks]