King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/MP36 Medical Imaging and Measurement

Summer 2001

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

Separate answer books must be used for each Section of the paper.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

SECTION A: Answer SIX parts of this section

1.1) Describe how the physics of the interaction between photons and tissue influences the way in which an x-ray mammography system is developed.

[7 marks]

1.2) An imaging system has a point spread function of the form

$$h(x,y) = A \text{ for } |x| \le a \text{ and } |y| \le a$$
$$= 0 \text{ for } |x|,|y| > a$$

where A is a constant. Calculate the modulation transfer function of the system.

[7 marks]

1.3) State the advantages and disadvantages of using Tc-99m as a radionuclide for SPECT scanning.

[7 marks]

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1.4) A radiographer is planning to use an ultrasound system to obtain a 100-line pulse-echo B-mode image to a depth of 15 cm in liver. What is the maximum pulse repetition frequency and frame rate at which the system can be operated? (The speed of sound in liver is 1500 m s^{-1}.)

[7 marks]

1.5) Define the terms *characteristic acoustic impedance* and *specific acoustic impedance*. Show that the particle velocity, u, and acoustic pressure, p, are in phase for a forward travelling, single frequency, plane wave $p(x,t) = A\sin(\omega t - kx)$. Derive an expression for the intensity of this wave.

[7 marks]

1.6) Describe how, in an appropriate magnetic resonance system, the free induction decay (FID) of the magnetisation of the protons in a tissue *slice* through a patient can be measured.

[7 marks]

1.7) The point spread function of an imaging system is isotropic, and is given by $h(|\vec{r}|)$. Show that the Fourier transform of the image of a two-dimensional object, $O(x, y)$, is given by the product of the Fourier transform of the object and the optical transfer function of the system.

[7 marks]

1.8) A flat disc of radius R and uniform density C lies in the (x, y) plane. Calculate the density projection of the disc onto the direction of the x-axis.

[7 marks]

SECTION B: answer TWO questions

2) (a) A planar interface separates two media of different densities but in which the speed of sound is the same. Derive an expression for the ultrasound amplitude reflection coefficient for plane waves incident normally on the interface.

.[8 marks]

(b) What is meant by the pulse-echo principle used in ultrasound imaging? List the assumptions made when implementing this technique. Explain how this principle is employed in the design of a linear multi-element array imaging device.

[12 marks]

(c) Write down the Doppler equation describing the frequency shift of ultrasound back-scattered from moving blood. An ultrasound wave of frequency 5 MHz is scattered from red cells in blood moving away from the ultrasound source at a speed of 30 cm s^{-1}. The blood vessel makes an angle of 60° with the beam. What is (i) the frequency and (ii) the wavelength of the ultrasound reflected back to the source?
(The speed of sound in blood and in the ambient soft tissue is 1500 m s^{-1}.)

[10 marks]

3) (a) Images are obtained using a gamma camera with a pinhole collimator. With the aid of a diagram, use geometric arguments to indicate qualitatively how the magnification and resolution of the system depend on the distance between the aperture and the source.

[8 marks]

(b) With the aid of a labelled diagram, discuss the basic principles of the operation of a typical modern PET scanner. Explain how fundamental features of electron/positron annihilation are exploited in the design of such a scanner and indicate those factors which influence its image resolution.

[12 marks]

(c) Show that the linear attenuation coefficient in soft tissue of a γ-photon emitted by a Tc-99m nucleus is approximately proportional to the density of the tissue.

[10 marks]

4) (a) A proton is placed in a constant magnetic field, $\vec{B}$, with its magnetic moment, $\vec{\mu}$, initially at an angle, θ, to the field. Show that $\vec{\mu}$ precesses about the direction of $\vec{B}$ and obtain an expression for the magnitude of the Larmor frequency.
{The vector identity $(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$ may be useful.}

[20 marks]

(b) Derive an expression for the fractions of ^{1}H nuclei in the upper and lower energy states in a magnetic field of B tesla at a temperature of $T\,^\circ$C. How long will it take for the magnetisation to relax to 5% of its equilibrium value at B tesla and $T\,^\circ$C when the field is switched off, if the longitudinal relaxation time is known?

[10 marks]

5) (a) Explicitly validate the Fourier Slice Theorem for any projection of a two-dimensional object distribution, $D(x, y)$. How is this theorem used in x-ray CT scanners?

[12 marks]

(b) An x-ray system is known to have an isotropic point-spread function. Derive the relationship between the point-spread and line-spread functions, by calculating the image of a line object placed in any convenient orientation.

[10 marks]

(c) Obtain an expression for the maximum kinetic energy, K, of the recoil electron in a Compton event occurring in the x-ray beam generated by a diagnostic mammography unit operated at V kVp. Show that K is proportional to V^2 to good approximation.

[8 marks]