King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/MP36 Medical Imaging and Measurement

Summer 2000

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in s quare brackets.

Separate answer books must be used for each Section of the paper.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Planck constant h = 4.14×10^{-15} eV s
Gyromagnetic ratio of proton γ = 4.26×10^{7} s^{-1} T^{-1}

SECTION A - Answer SIX parts of this section.

1.1) Calculate the projection onto the x-axis of a two-dimensional, planar, circular object with constant 'density', C, and radius R.

[7 marks]

1.2) Describe those features of x-radiation, and its propagation in human tissues, that make it suitable for medical computerised tomography.

[7 marks]

1.3) Obtain an expression for the energy of the *softest* photon able to impart a kinetic energy, W, to a recoil electron in a Compton scattering event. Assume that non-relativistic conditions apply for the recoil electron.

[7 marks]

1.4) What is, approximately, the speed of sound in soft tissue? For diagnostic purposes, an ultrasound B-mode (pulse-echo) image to a depth of 20 cm in liver is required. What is the maximum pulse repetition frequency at which the system can be operated? What image frame rate is possible if 120 image lines per frame are required?

[7 marks]

1.5) Derive an expression for the amplitude reflection coefficient of an ultrasound wave passing from muscle into liver at normal incidence across a planar interface located at x=0. The two tissues have the same density, ρ , but propagate ultrasound waves at different speeds, V_1 and V_2.

[7 marks]

1.6) Discuss briefly the advantages and disadvantages of using the radionuclide Tc99m for single photon emission computed tomography (SPECT) scanning.

[7 marks]

1.7) Describe how the free induction decay (FID) of the magnetisation of a small tissue volume could be measured using appropriate magnetic resonance apparatus. How should the measurement scheme be extended to determine the longitudinal relaxation time of the volume.

[7 marks]

1.8) An imaging system has a point spread function that is constant for $|\boldsymbol{r}| < \mathrm{R}$ and 0 otherwise, where $\boldsymbol{r}$ is the position vector. Obtain an expression for the modulation transfer function (MTF) for the system.

[7 marks]

SECTION B - Answer TWO questions

2) (a) A narrow, monochromatic beam of x-rays is incident on a thin layer of material, containing N atoms per cm^3. The interaction of each atom with the photons is expressed by the total cross section, σ cm^2. Derive expressions for (a) the linear attenuation coefficient and (b) the mass attenuation coefficient of the material in terms of N and σ. Be explicit about any approximations made, and include the dimensions of the derived quantities.

[8 marks]

(b) An x-ray system is known to have an isotropic point-spread-function. Show how the point-spread-function can be calculated from an analysis of the image of the straight edge of a metal plate.

[12 marks]

(c) Explicitly validate the Fourier Slice Theorem for any projection of the two-dimensional object distribution $D(\boldsymbol{r}) = \exp(-k|\boldsymbol{r}|^2)$, where $\boldsymbol{r}$ is the position vector and k is a constant.

[10 marks]

3) (a) Explain, with appropriate analysis, one procedure by which the Doppler shift in a continuous wave Doppler device may be recovered.

[10 marks]

(b) What is the maximum detectable speed of movement in soft tissue, at a depth of 5 cm, using a pulse-wave Doppler device with variable pulse repetition frequency (PRF) and an ultrasound operating frequency of 5 MHz?

[12 marks]

(c) Show that the intensity of a plane ultrasound pressure wave of the form $p(x,t) = A\sin(kx - \omega t)$ is proportional to the square of its amplitude.

[8 marks]

4) (a) Compare positron-emission tomography (PET) and single-photon-emission- computed tomography (SPECT) with respect to the following characteristics: principles of data collection, measured input to image recovery, radionuclides used, spatial resolution, accuracy of attenuation correction.

[10 marks]

(b) The cross-section for Compton scattering of γ -ray photons in soft tissue is experimentally determined to be constant in a particular energy range. Show that, in this energy range, the mass attenuation coefficient for γ -ray photons in soft tissue is approximately constant. (You may assume that Compton scattering is the dominant interaction in the energy range considered.)

[10 marks]

(c) Show how an isotropic optical transfer function for a gamma camera can be calculated from the image of a thin, straight, radioactive wire.

[10 marks]

5) (a) A proton is placed in a constant magnetic field, ***B***, with its magnetic moment, μ , oriented at an angle, θ , to the field direction. Show that the magnetic moment precesses about the direction of the field.

[10 marks]

(b) Find the energy difference between the low and high energy states of a proton in a 1 tesla magnetic field, given that the resonance frequency for a proton in a 7 tesla field is 300 MHz.

[6 marks]

(c) In the 1.5 tesla constant magnetic field of a magnetic resonance imaging (MRI) unit, the separation of the main spectral peak due to water and that due to fat is approximately 3 ppm. Estimate the frequency shift in hydrogen between water and fat.

[4 marks]

(d) Explain how a tissue sample placed in the magnetic field can acquire a *constant* magnetisation directed along the field direction. Describe how, in a typical MRI scanner, the direction of the magnetisation may be temporarily tipped through $\pi/2$, and the subsequent return to equilibrium monitored.

[10 marks]

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