

# King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

**B.Sc. EXAMINATION**

**CP/MP36 Medical Imaging and Measurement**

**Summer 2003**

**Time allowed: THREE Hours**

**Candidates must answer SIX parts of SECTION A  
and TWO questions from SECTION B**

**The approximate mark for each part of a question is indicated in square brackets**

**You must not use your own calculator for this paper  
Where necessary, a College Calculator will have been supplied**

**TURN OVER WHEN INSTRUCTED**

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**SECTION A - Answer SIX parts of this section.**

- 1.1) An imaging system has a point spread function expressible as  

$$P(x, y) = A \quad \text{for } |x| \leq x_0, |y| \leq y_0$$

$$= 0 \quad \text{otherwise}$$
 where  $A$  is a constant. Calculate the modulation transfer function for this system. [7 marks]
- 1.2) Describe a "first generation" X-ray system for medical computerised tomography. List those features of X-radiation, and its propagation in human tissues, that make it suitable for this type of imaging. [7 marks]
- 1.3) Obtain an expression for the intensity of the ultrasound wave  
 $p(x, t) = A \sin(2\pi\{ft - x/\lambda\})$ , where  $A$  is the (constant) amplitude,  $f$  is the frequency and  $\lambda$  is the wavelength of the wave. [7 marks]
- 1.4) Describe how, in an appropriate magnetic resonance system, the free induction decay (FID) of the magnetisation of a tissue *slice* through a patient can be measured. [7 marks]
- 1.5) Discuss why Tc-99m is a suitable radionuclide to use for single photon emission tomography (SPECT) scanning. Also, indicate any disadvantages it has for medical imaging? [7 marks]
- 1.6) A narrow, monochromatic, beam of X-rays is incident on a thin layer of material, consisting of  $N$  atoms per  $\text{cm}^3$ . The interaction of each atom with the photons is expressed by the total cross section,  $\sigma \text{ cm}^2$ . Derive expressions for the material's linear attenuation coefficient and mass attenuation coefficient in terms of  $N$  and  $\sigma$ . [7 marks]
- 1.7) Calculate the projection (onto any direction in the plane of  $\mathbf{r}$ ) of an annular shaped disk with density,  

$$D(\mathbf{r}) = C \quad \text{for } a \leq |\mathbf{r}| \leq b$$

$$= 0 \quad \text{otherwise}$$
 where  $C$  is a constant. [7 marks]
- 1.8) Sketch and name three types of collimator, other than the pinhole collimator, used with gamma cameras and indicate how the image relates to the object. [7 marks]

## SECTION B - answer TWO questions

- 2) a) Plane waves of ultrasound are incident normally at a planar interface between two media with different characteristic acoustic impedances. Derive expressions for the ultrasound amplitude reflection and transmission coefficients of these waves.

[8 marks]

- b) A 5 MHz ultrasound real-time pulse-echo B-mode system is operated with a pulse repetition frequency (PRF) of 2.5 kHz.

- i) To what depth in soft tissue can images be formed?

[4 marks]

- ii) What is the maximum frame rate if 125 line images are desired?

[2 marks]

- iii) If the system is operated in Doppler mode, what is the maximum ('line-of-sight') speed of movement that can be detected within the imaged region?

[6 marks]

Assume that the speed of sound in soft tissues is  $1500 \text{ m s}^{-1}$ .

- c) Briefly describe the processes of envelope detection, time gain compensation, compression and (line) interpolation. State why these processes are incorporated in medical ultrasound B-mode image formation.

[10 marks]

- 3) a) A single photon emission tomographic (SPECT) system is used to image a bounded region with uniform (radio-) activity enclosed within a patient containing no other activity.

- i) Assume that the linear attenuation coefficient for  $\gamma$  - photons of the appropriate energy is constant everywhere within the patient to develop an approximate attenuation correction procedure to be applied to the measured projection data.

[15 marks]

- ii) What additional information is needed in order to carry out the attenuation correction, and how would it be obtained in practice?

[3 marks]

- b) The point spread function (PSF) of the imaging system in a) is measured to be of the shape

$$\text{PSF}(x, y) = A \cdot \exp(-a|x|) \cdot \exp(-a|y|) \text{ with } a \text{ and } A \text{ constants.}$$

Calculate:

- i) the line spread function (LSF) of the system

[4 marks]

- ii) the edge spread function (ESF) of the system

[5 marks]

- iii) the phase transfer function (PTF) of the system

[3 marks]

- 4) a) A proton is placed in a constant magnetic field,  $\mathbf{B}$ , with its magnetic moment,  $\mu$ , initially at an angle,  $\phi$ , to the field.
- Show that  $\mu$  precesses about the direction of  $\mathbf{B}$ . [8 marks]
  - Obtain an expression for the magnitude of the Larmor frequency. [10 marks]
- Hint: Use classical arguments and note the following vector identity,  
 $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{b}(\mathbf{c} \cdot \mathbf{a}) - \mathbf{a}(\mathbf{b} \cdot \mathbf{c})$
- Derive an expression for the fractions of  $^1\text{H}$  nuclei in the upper and lower energy states, in a constant magnetic field of magnitude  $B$  tesla at temperature  $T$  °C. [9 marks]
  - In a magnetic resonance spectrum, the separation of the main spectral peak due to water and the peak due to lipids is approximately 3 ppm. If the static magnetic field strength of a magnetic resonance imaging unit is 1.5 tesla, what is the frequency shift in hydrogen between water and lipid? [3 marks]
- 5) a) State the Fourier Slice Theorem and explicitly validate it for any projection of a two dimensional density function,  $D(x,y)$ . [10 marks]
- An X-ray beam is generated by a diagnostic mammography unit operating at a peak voltage of  $V$  kV. Obtain an expression for the maximum kinetic energy,  $K$ , of a recoil electron in a Compton event produced by these X-rays. Show that  $K$  is proportional to  $V^2$ , to reasonable approximation. Note that the rest mass of an electron is  $\sim 500$  keV. [10 marks]
  - Describe how the design of an X-ray mammography unit is determined by the physics of the photon / tissue interaction. [10 marks]