King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/MP33 Medical Engineering

Summer 1999

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

SECTION A - Answer SIX parts of this section.

1.1) Show how a suitable arrangement of ECG leads may be used to measure the instantaneous spatial orientation of the activity vector of the heart. You may assume that the electrical action of the heart is represented by an activity vector of strictly dipole character.

[7 marks]

1.2) Starting from the equation of continuity, derive Fick's second law for diffusion processes in one dimension. Clearly indicate any significant assumptions that are made.

[7 marks]

1.3) The time-averaged speed of blood flow is measured to be 20 cm s^{-1} in the aorta (close to the heart) and 0.5 mm s^{-1} in the capillaries. The aortic diameter is approximately 3000 times that of a typical capillary. Use this information to make a rough estimate of the number of capillaries in the body. What additional information is needed to calculate the cardiac output in this case?

[7 marks]

1.4) Use Bernouilli's law to substantiate the empirical finding that the pressure drop (in mm Hg) across a stenosis is approximately four times the square of the velocity (in m s^{-1}) of the blood jet through the blockage. (1 atm ≈ 75 cm Hg ≈ 10^5 pascals).

[7 marks]

1.5) The arterial and venous concentrations of oxygen are measured directly from blood samples, and the rate of oxygen consumption by the body is measured with a spirometer. Show how these measurements may be used to estimate cardiac output.

[7 marks]

1.6) Compare and contrast two methods of temperature measurement used in medicine, and based on different temperature sensors, paying attention to the advantages and disadvantages of the transducers used.

[7 marks]

1.7) The resistivity of a cell membrane is determined by measuring the decay of the leakage current across it. Develop a simple model which expresses the resistivity in terms of experimentally accessible data. Be specific about the approximations made, and list what other data are needed in order to obtain a numerical value for the resistivity in this experiment.

[7 marks]

1.8) Define what is meant by *a discrete time signal* and *a digital signal*. For the continuous time signal $x(t) = A\cos(\Omega t)$ as Ω increases $x(t)$ oscillates more rapidly. Describe the behaviour of the discrete time signal $x[n] = A\cos[\omega n]$ with varying ω and comment on the interpretation of 'high' and 'low' frequency.

[7 marks]

SECTION B - answer TWO questions

2) The major constituents of the normal electroencephalogram (EEG) are the alpha wave centred around 10 Hz and the beta wave which occurs around 18 Hz. If the EEG signal is sampled at 100 samples per second, what are the highest harmonics of the alpha and beta waves that may be reliably observed without aliasing?

[6 marks]

For a data acquisition system suitable for the above application, write short notes on the influence and relevance of the following factors:

i) Input signal conditioning, including dynamic range considerations.
ii) Speed, bearing in mind requirements for multiplexing and aliasing.
iii) Accuracy, with particular reference to resolution and calibration.

[9 marks]

Give definitions for a linear system and a time-invariant system.

[4 marks]

By expressing an input signal in the form $x[n] = \sum_{-\infty}^{\infty} x[k]\delta[n-k]$ derive the expression for the output of a linear time invariant system $y[n] = \sum_{-\infty}^{\infty} x[k]h[n-k]$ where $h[n]$ is the impulse response.

[8 marks]

Show that the complex exponential sequence $x[n] = \exp(i\omega n)$ is an eigenfunction of a linear time-invariant system.

[3 marks]

3) What is meant by a viscoelastic material? Write down the pressure/density relation in the following two types of material, and find the analytical forms of the associated relaxation and creep responses in each:

i) an elastic material;
ii) a Kelvin-Voigt material (mechanical element symbolically denoted by a spring and a dashpot in parallel);

[10 marks]

The mechanical behaviour of a tissue is modelled as a simple Kelvin-Voigt material. Using Newton’s second law and the continuity equation, derive a one-dimensional equation describing the (linear) propagation of pressure waves in the tissue, assuming that it is both uniform and isotropic. In what way would this model be inadequate for describing pressure waves in muscle tissue?

[12 marks]

Demonstrate the principles of impedance plethysmography by showing how volume changes caused by blood flow pulsations in an arm may be estimated by measuring the electrical impedance of the limb. Point out any assumptions that are made in the method.

[8 marks]

4) Blood flow in a vessel is modelled as a Newtonian fluid flowing in a rigid pipe (of constant circular cross section), and driven by a constant pressure gradient. In your answers, be explicit about any assumptions made in the model.

(a) Show that the velocity profile is parabolic.

[12 marks]

(b) Show that the volume flow rate depends on the fourth power of the tube radius.

[3 marks]

(c) Derive a relationship between the mean and maximum velocities of the fluid.

[3 marks]

(d) In what way would the model have to be modified in order to make it more realistic for describing flow in an artery?

[4 marks]

Derive a statement of Laplace's law by considering the tensile stress, σ, in the walls of a spherical shell, radius R, with internal pressure P. Indicate how this law predicts the dependence of wall thicknesses on the internal diameter of blood vessels.

[8 marks]

5) Construct a simple model for a renal dialysis system. Treat all the body fluid (including blood) as one subsystem, separated via a semi-permeable membrane from the dialysis fluid, considered as the second subsystem. Make reasonable assumptions to obtain an expression for the time-dependence of the concentration of toxic substances in the blood. What do your results tell you about the design of a dialysis system?

[12 marks]

Consider two compartments separated by a membrane with an electrical potential difference, V, across it, and through which ions can pass freely. Express the ratio of ion concentrations in the two compartments in terms of V and the temperature, T, at thermodynamic equilibrium (Nernst equation).

[6 marks]

Show that the electrostatic potential outside an activated long cylindrical cell (such as an axon) immersed in an infinite, uniform, conducting medium may be expressed in terms of the second derivative of the interior potential. State clearly any approximations made.

[12 marks]