# King's College London

## UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

### **B.Sc. EXAMINATION**

CP/MP33 Medical Engineering

Summer 1998

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College Calculator will have been supplied.

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#### **CP/MP33**

[7 marks]

#### **SECTION A - Answer SIX parts of this section.**

- 1.1) Calculate the cardiac output of a patient for whom the following data are known: oxygen consumption, 250 ml/min; venous oxygen content, 0.15 ml/ml; arterial oxygen content, 0.20 ml/ml.
- 1.2) Starting from Bernoulli's equation, show how the area of the aortic valve may be estimated roughly from measurements of the patient's aortic pressure, left-ventricular pressure and cardiac output.

1.3) Explain what is meant by each of the following terms in the context of the digitisation of an analog signal: *sampling*, *Nyquist frequency*, *aliasing*, and *quantisation errors*.[7 marks]

1.4) Use a simple model to show that the rate of decay of the voltage across a membrane is independent of the thickness of the membrane.

1.5) Derive Fick's second law of diffusion, stating the assumptions made.

1.6) Define *positive feedback*, *negative feedback* and *feedforward control*. Give an example of biological feedforward control. Explain why most control systems in the body operate via negative feedback.

1.7) Show how the rate of blood flow in the forearm may be determined from measurements of the electrical impedance of the limb. Indicate any assumptions that are made in the calculation.

1.8) What are the advantages and disadvantages of thermocouple devices for temperature measurement in physiological monitoring?

[7 marks]

[7 marks]

[7 marks]

[7 marks]

[7 marks]

[7 marks]

#### **SECTION B - answer TWO questions**

2) A renal dialysis arrangement is modelled as a simple two-compartment system. Treat the blood as one compartment, with the dialysis fluid as the second. Make reasonable assumptions to obtain an expression for the time-dependence of the concentration of impurities in the blood. Show that the time-constant describing this behaviour is inversely proportional to the surface area of the dialysis membrane. What do your results tell you about the design of a dialysis system?

Assume that the electrical action of the heart is represented by a single activity vector of dipole character. Show how a suitable arrangement of ECG leads may be used to measure the time-dependence of the spatial orientation of the activity vector.

A linear axon lies along the *x*-axis. As it depolarises, a constant electrical activity vector, p, sweeps along the cell with speed u. An electrode at x = 0, y = a measures the potential, V. Find an expression for the measured potential as a function of time, t, and sketch its shape. Assume that at t = 0, p is directly under the electrode.

[8 marks]

[14 marks]

[8 marks]

**3**) A simple model for blood flow in a vessel is that of a Newtonian fluid flowing in a rigid circular pipe (of constant cross-section), and driven by a constant pressure gradient. Show that the velocity profile is parabolic. Be explicit about any assumptions that are made. Derive an expression for the variation of the shear stress with position in the fluid, as well as for the volume flow rate.

[14 marks]

Blood behaves as a visco-plastic fluid, rather than as a Newtonian fluid. Discuss, qualitatively, how the flow profile in the above model is affected if blood is assumed to be an ideal Bingham plastic.

[6 marks]

Assuming a constant pressure gradient, find an expression relating the mean and maximum velocities of a Newtonian fluid flowing in a rigid circular pipe. How is this relation altered when the fluid is a Bingham plastic?

[10 marks]

#### 4) Define the terms: *analog*, *discrete*, and *digital* signals.

[4 marks]

Define the terms: *memoryless, linearity, time invariance* and *causality*, as applied to a digital system. A digital system which has the properties of linearity and time invariance has an impulse response h[n] and frequency response  $H(\omega)$ . Show that the response to the input sequence  $x[n] = \exp(j\omega n)$  is the output sequence  $y[n] = H(\omega)\exp(j\omega n)$ , and comment on the result.

[12 marks]



Give expressions for the open loop gain,  $G_1$  and  $G_2$  for the feedback system shown in the diagram.

[3 marks]

Derive expressions for the change in x with feedback present in terms of the open loop gain and the change in x without feedback present. Comment on the results.

[11 marks]

5) Experiments indicate that muscle tissue may be modelled as a simple Maxwell (viscoelastic) solid. Describe what this implies, in terms of the relationship between applied pressure and density changes in the muscle.

[4 marks]

Use the above result, together with Newton's second law and the continuity equation, to derive a linear one-dimensional equation describing the propagation of pressure waves in the muscle. You may assume that the tissue is uniform and isotropic.

[14 marks]

Show how the visco-elastic parameters of the tissue model may be obtained from measurements of the propagation velocity and attenuation of pressure waves in the muscle.

[12 marks]

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