King’s College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CPMP33 Medical Engineering

Summer 2004

Time allowed: THREE Hours

Candidates should answer no more than SIX parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	e_0	=	8.854×10^{-12}	F m^{-1}
Permeability of free space	m_0	=	$4\,\pi \times 10^{-7}$	H m^{-1}
Speed of light in free space	c	=	2.998×10^{8}	m s^{-1}
Gravitational constant	G	=	6.673×10^{-11}	N m^{2} kg^{-2}
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
		=	4.136×10^{-15}	eV s
Boltzmann constant	k_B	=	1.381×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	s	=	5.670×10^{-8}	W m^{-2} K^{-4}
Gas constant	R	=	8.314	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.022×10^{23}	mol^{-1}
Molar volume of ideal gas at STP		=	2.241×10^{-2}	m^{3}
One standard atmosphere	P_0	=	1.013×10^{5}	N m^{-2}

SECTION A - Answer SIX parts of this section

1.1) Derive the continuity equation $\frac{\partial C}{\partial t} = -\frac{\partial j}{\partial x}$ which links the number flux density, j, to the particle concentration, C, for a one dimensional flow of particles.

[7 marks]

1.2) An impedance plethysmography system is used to measure blood flow. Show that the change in electrical impedance, Z, created by the pulsatile nature of blood flow in the forearm is given by $-\frac{Z^2}{Z+Z_B}$, where Z_B is the shunting impedance of the extra blood flowing into the limb during a heart beat.

[7 marks]

1.3) Derive Laplace's principle for the case of a thin-walled spherical vessel. Explain how this principle leads to a possible understanding of the relative thickness of the walls of arteries of different diameters.

[7 marks]

1.4) Derive an expression for the creep response of muscle tissue, if modelled as a simple Maxwell medium consisting of a viscous and elastic mechanical element in series.

[7 marks]

1.5) List seven factors to be considered when designing a surgical implant.

[7 marks]

1.6) Explain the indicator-dilution method for measuring blood flow, and show how it can be used to derive an expression for cardiac output in terms of the arterial and venous concentrations of oxygen, and the rate of oxygen consumption by the body.

[7 marks]

1.7) Use Bernoulli's theorem to justify the cardiological rule-of-thumb that the blood pressure drop Δp (in mm Hg) across a stenosis is four times the square of the speed (in m s^{-1}) of the blood jet through the blockage.
Note that: 1atm = 10^5 pascals $\approx$750 mm Hg.

[7 marks]

1.8) Show how a continuous wave Doppler instrument can recover the Doppler shift in the reflection signal from moving blood, by the method of coherent demodulation.

[7 marks]

SECTION B - Answer TWO questions

2)a) A collection of spherical particles, each of radius a, is suspended in a liquid with fluid viscosity η at temperature T. Show that $D = k_B T / 6\pi\eta a$ (Einstein's relationship), where D is the diffusion coefficient and k_B is Boltzmann's constant. Assume that the magnitude of the viscous force experienced by the particles is given by $6\pi\eta a v$, with v the particle speed (Stokes' law).

[10 marks]

b) Diffusion of particles takes place across a planar boundary, where the following boundary conditions hold:

$$C(x,t) = C_0 \text{ for } x = 0, \quad t > 0$$

$$C(x,t) = 0 \quad \text{for } x > 0, \quad t = 0$$

where $C(x,t)$ denotes the particle concentration at location x at time t, and C_0 is a constant. Use Boltzmann's method to obtain an integral expression for $C(\xi)$ taking $\xi = x/\sqrt{4Dt}$ in the diffusion equation, where D is the diffusion coefficient. Assume that C is a function of ξ only.

[12 marks]

c) A renal dialysis machine is modelled as a simple two-compartment system, with the blood and its impurities as one compartment and the dialysis fluid as the second. Obtain an expression for the time-dependence of the concentration of impurities in the blood, stating the assumptions made, and show that the time-constant describing this process is inversely proportional to the surface area of the dialysis membrane.

[8 marks]

3)a) Blood flow in a vessel is modelled as a viscous, incompressible Newtonian fluid flowing in a smooth, rigid circular pipe of radius R, and subjected to a constant pressure gradient Δp. Show that the velocity profile, $u(r)$, of the flowing blood is $\frac{\Delta p}{4\eta}(R^2 - r^2)$, where η is the coefficient of viscosity and r is the distance from the central axis of the pipe.

[14 marks]

b) Use the result in a) to show that the mean flow speed, $\overline{V}$, is equal to half the maximum blood speed, V_{MAX}, in the vessel, and thus express the shear stress exerted by the blood on the vessel wall in terms of $\overline{V}$.

[12 marks]

c) Give four reasons why the above model is unrealistic.

[4 marks]

4) The mechanical behaviour of liver tissue is modelled as a uniform, isotropic Kelvin-Voigt medium, which may be represented as a viscous and an elastic element in parallel.

a) Derive a relationship between stress and strain for such a medium, and interpret this to show that the constitutive equation for the relationship between changes in pressure and density in the model is $p = \kappa\varrho + \eta\frac{\partial\varrho}{\partial t}$, where p is the pressure, ϱ is the density, and κ and η are constants.

[8 marks]

b) Assume that the (mass) continuity equation holds in this tissue and that the force equation contains only pressure forces operating in accordance with Newton's second law. Derive appropriate forms of these two equations, and combine them with the constitutive equation to obtain a wave equation for the linear propagation of pressure waves in liver tissue.

[14 marks]

c) Derive the analytical form of the stress/strain creep response for the above tissue model.

[8 marks]

5)a) An electrical impulse passes along a cylindrical axon of radius a, immersed in a medium of uniform conductivity σ_o. Show that the depolarisation front generates an instantaneous electrostatic potential at a distant exterior point, $\boldsymbol{R}$, that is given by

$$V_o(\boldsymbol{R}) = (\frac{a^2\sigma_i}{4\sigma_o})\frac{1}{R}\int_{x_1}^{x_2}\frac{\partial^2 V_i}{\partial x^2}\left(1+\frac{x}{R}\cos\theta\right)dx$$

with V_i the interior potential, σ_i the interior conductivity, and with the limits of the depolarisation front located at x_1 and x_2. θ is the angle between the x-axis (along the axon) and $\boldsymbol{R}$. The origin of the co-ordinate system is located within the depolarisation front.

[14 marks]

b) The resistivity of a cell membrane is assessed by measuring the decay of its leakage current. By considering the membrane to be a simple capacitor, express the resistivity in terms of the data recorded in the experiment. What other data need to be known in order to obtain a numerical value for the resistivity?

[8 marks]

c) A solution is placed into a receptacle which is divided into two compartments by a membrane which allows only a particular ion species to pass freely through. A potential difference V is maintained between the two compartments. Obtain an expression for the ratio of the concentrations of the ion species in the two compartments at thermodynamic equilibrium (Nernst equation), as a function of temperature and the valence of the ion.

[8 marks]