SECTION A: Answer SIX parts of this section

1.1) Derive an expression for the cardiac output in terms of the arterial and venous concentrations of oxygen and the rate of oxygen consumption by the body. Calculate the cardiac output of a patient for whom the following data are known: oxygen consumption, 250 ml/min; venous oxygen content, 0.15 ml/ml; arterial oxygen content, 0.20 ml/ml.

[7 marks]

1.2) Starting from Bernoulli's equation, show how the area of the aortic valve may be estimated from measurements of a patient's aortic and left-ventricular pressures, cardiac output, ejection period and heart rate.

[7 marks]

1.3) A pulse of x-rays of cross-section 1500 cm^2 consists of 10^{13} photons, each with energy 40 keV, and is switched on for 0.1 seconds. Calculate the photon flux, photon flux density, energy flux, and intensity of the beam.

[7 marks]

1.4) Derive, from first principles, the (one-dimensional) continuity equation which links number flux density to particle concentration.

[7 marks]

1.5) An ultrasound wave of frequency 5 MHz is scattered from red cells in blood moving directly away from an ultrasound source at a speed of 50 cm s^{-1}. What is the wavelength of the ultrasound reflected back to the source?
Assume that the speed of sound both in the ambient soft tissue and in blood is 1500 m s^{-1}.

[7 marks]

1.6) List seven factors that must be considered when designing a surgical implant.

[7 marks]

1.7) Show how the one-dimensional diffusion equation $\frac{\partial C}{\partial t} = D(t)\frac{\partial^2 C}{\partial x^2}$ with a time-dependent diffusion coefficient, $D(t)$, can be reduced to one with a constant diffusion coefficient.
In the equation, C is concentration and x is position.

[7 marks]

1.8) Calculate the creep response of a simple Kelvin-Voigt medium for which the dependence of stress, s, and strain, e, are related by $s = ae + b\dot{e}$, where a and b are constants.

[7 marks]

SECTION B: answer TWO questions

2) a) Use a simple model of an axon to obtain an integral expression for the electrostatic potential outside the depolarisation front in a long cylindrical cell, in terms of the second derivative of the interior potential. State clearly any approximations made.

[10 marks]

b) An attempt is made to assess the resistivity of a cell membrane by measuring the decay of the leakage current across it. Representing the membrane by a parallel-plate capacitor that is discharging, express the resistivity in terms of the data recorded in the experiment. What other data need to be known in order to obtain a numerical value for the resistivity?

[10 marks]

c) Blood flow in a leg may be determined from measurements of changes in its electrical impedance arising from pulsatile blood flow. Derive an expression for the change in impedance of the leg as it dilates due to an increased volume of blood.

[10 marks]

3) a) A simple model for blood flow in a vessel is that of a Newtonian fluid flowing in a rigid circular pipe of constant cross section, and driven by a constant pressure gradient. Show that in this case the velocity profile is parabolic. Derive an expression for the variation of the shear stress with position in the fluid.
Give three major reasons why such a model is unrealistic.

[20 marks]

b) Calculate the volume flow rate in the above mode, and show that the mean and maximum velocities, u_{mean} and u_{max}, are related by $u_{mean} = \frac{1}{2} u_{\max}$.

[10 marks]

4) a) State Fick's second law of diffusion. Show that if a concentration of particles is initially Gaussian in position, with variance $s^2(t=0)$, it will remain distributed as a Gaussian after time t, centred on the same point, but with a larger variance, which depends linearly on t.

Hint: Try the trial solution $C(x,t)=\left(\frac{A}{s}\right)\exp\left(-\frac{x^2}{2s^2}\right)$ in the diffusion equation.

[14 marks]

b) A renal dialysis machine is modelled as a simple two-compartment system. Treat the blood as one compartment, with the dialysis fluid as the second. Making reasonable assumptions, which should be stated, obtain an expression for the time-dependence of the concentration of impurities in the blood.
Show that the time-constant describing this behaviour is inversely proportional to the surface area of the dialysis membrane.
What do your results tell you about the design of a dialysis system?

[16 marks]

5) a) Muscle may be modelled as a Maxwell medium. Calculate the stress-strain relationship for a Maxwell material, with its mechanical element symbolically denoted by a spring and dashpot in series.
Indicate how to extend this result to suggest how density changes are influenced by pressure changes in such a medium.

[8 marks]

b) Use Newton's second law and the continuity equation to show that the propagation of pressure waves in the muscle may be described by the equation: $\frac{\partial^2 p}{\partial x^2}-a\frac{\partial^2 p}{\partial t^2}-b\frac{\partial p}{\partial t}=0$,
where p denotes the pressure and a and b are constants.
You may assume that the tissue is uniform and isotropic.

[14 marks]

c) Show schematically how the the visco-elastic parameters of the above tissue model may be related to measurements of the (dispersive) propagation velocity and attenuation of pressure waves in the muscle.

[8 marks]

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