King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/MP33 Medical Engineering

Summer 2001

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

Separate answer books must be used for each Section of the paper.

**You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.**

TURN OVER WHEN INSTRUCTED

SECTION A: Answer SIX parts of this section

1.1) Show that a one-dimensional diffusion equation with a time-dependent diffusion coefficient can be reduced to an equation with a constant diffusion coefficient.

[7 marks]

1.2) The concentration of particles is observed to be a Gaussian distribution, centred on the origin. Obtain an expression for the number flux density resulting from diffusion. What is the mass flux density if 25% of the particles have mass m_1 and the rest have mass m_2? You need work only in one dimension, and may assume that the diffusion process is independent of particle mass.

[7 marks]

1.3) Use simple analysis to explain one procedure by which the Doppler shift in a continuous wave Doppler device may be recovered.

[7 marks]

1.4) Use Bernoulli's theorem to justify the cardiologists' rule-of thumb that the pressure drop (in mm Hg) through a stenosis is approximately four times the square of the speed (in m s^{-1}) of the blood jet through the blockage.
(1 atm ≈ 10^5 Pa ≈ 750 mm Hg)

[7 marks]

1.5) Derive Laplace's principle for the case of a long thin-walled cylinder. Indicate how this principle may be used to explain the different thicknesses of blood vessel walls in the human circulatory system.

[7 marks]

1.6) Describe a simple electrical circuit which can be used to model the basic behaviour of a cell membrane. Use the model to show that the rate of decay of the voltage across a membrane is independent of its thickness.

[7 marks]

1.7) The time-averaged speed of blood flow in the aorta is measured to be about 400 times greater than that in the capillaries. The diameter of the aorta is approximately 3200 times that of a typical capillary. Use this information to make a rough estimate of the number of capillaries in the body.

[7 marks]

1.8) What is meant by the *creep response* of a medium? Obtain an expression for the creep response of a Kelvin-Voigt medium.

[7 marks]

SECTION B: answer TWO questions

2) (a) Derive, from first principles, the one-dimensional continuity equation which links the number flux density in a fluid to the particle concentration. Use this, and Fick's first law, to derive Fick's second law (the diffusion equation).

[10 marks]

(b) Diffusion of particles takes place across a planar boundary located at $x = 0$, where the following boundary conditions hold:

$$C(x,t) = C_0 \quad \text{for} \quad x = 0, \quad t > 0$$

$$C(x,t) = 0 \quad \text{for} \quad x > 0, \quad t = 0\,.$$

$C(x,t)$ denotes the particle concentration at location x and time t, while C_0 is a constant. Obtain an expression for the Laplace transform, with respect to the time variable, of the particle concentration for $x > 0$.

[12 marks]

(c) Obtain an expression for the time-dependence of the concentration of impurities in the blood when using a renal dialysis machine. Model the arrangement as a simple two-compartment system, with the blood and its impurities as one compartment, and the dialysis fluid as the second. Make reasonable assumptions to show that the time-constant describing the behaviour of the model is inversely proportional to the surface area of the dialysis membrane.

[8 marks]

3) (a) Given that the electrical activity of the heart may be described by a single dipolar activity vector, show that a precordial lead may be used to estimate the component of that vector orthogonal to the plane of the limb-lead components.

[10 marks]

(b) A linear axon lies along the x-axis. As it depolarises, a constant (dipole) electrical activity vector $\boldsymbol{p}$ sweeps along the cell with constant speed u. An electrode at $x = 0$, $y = a$ measures the potential, relative to zero at ∞. Find an expression for the potential, V, at the electrode as a function of time, t, and sketch it. Assume that at $t = 0$, $\boldsymbol{p}$ is directly under the electrode at $x = 0$. How would you incorporate the polarisation phase of the axon in this model?

[12 marks]

(c) A receptacle is divided into two identical compartments by a membrane which allows only one particular type of ion to pass freely through. The two compartments contain solutions with the same ionic composition. A potential difference, V volts, is maintained between the two compartments. Derive the Nernst equation, which relates V and the concentrations of the ion in the two compartments at thermodynamic equilibrium.

[8 marks]

4) (a) An artery may be represented by a long, cylindrical, elastic tube, isolated from its surroundings, with the blood considered as a homogeneous, incompressible, and inviscid liquid. Using this model, consider the case of a blood vessel branching into two identical 'daughters'. Obtain an expression for the reflection coefficient of a pressure wave, with amplitude p, from the junction. You may assume that, in this model, $\frac{p}{u} = \pm\rho\, c$, where u is the speed of flow ρ is the blood density and c is the pressure wave speed; the negative sign is used for a backward travelling wave. Under what conditions will there be no reflection from the junction?

[12 marks]

(b) Blood flow in an artery or vein may be modelled by the flow of a viscous Newtonian fluid through a rigid cylindrical tube, and subjected to a constant pressure gradient. Under these circumstances, the velocity profile of the flowing blood can be shown to be parabolic. Show that the volume flow rate depends on the fourth power of the tube radius, and that the spatial average (over the vessel cross section) of the flow speed is equal to half the maximum blood speed in the pipe. Give three major reasons why this model is unrealistic.

[12 marks]

(c) Use an indicator-dilution method to derive an expression for the cardiac output in terms of the arterial and venous concentrations of oxygen, and the rate of oxygen consumption by the body.

[6 marks]

5) (a) Skeletal muscle can be modelled as a uniform Maxwell medium, which may be represented as a viscous element and an elastic element in series. Derive the relationship between stress and strain for such a medium, and hence establish a constitutive equation for the dependence of pressure on density in the model. Assume that Newton's second law and the (mass) continuity equation for an elastic medium hold in this Maxwell model. Combine these with the constitutive equation to derive a wave equation for the propagation of linear pressure waves in skeletal muscle.

[22 marks]

(b) Derive the analytical form of the density-pressure creep response for skeletal muscle modelled as in (a). Illustrate your result with a simple diagram.

[8 marks]

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