King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/MP33 Medical Engineering

Summer 2000

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in s quare brackets.

Separate answer books must be used for each Section of the paper.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

$$1\,\text{atm} = 10^5 \text{ pascals} \approx 750\,\text{mm Hg}$$

SECTION A: Answer SIX parts of this section

1.1) Define the *number flux density* of a beam of particles. In a particle beam 20% of the particles have energy E_1, 45% have energy E_2, and the remainder have energy E_3; write down an expression for the intensity of the beam.

[7 marks]

1.2) Derive, from first principles, the one-dimensional continuity equation which links the number flux density to the particle concentration.

[7 marks]

1.3) Make some plausible assumptions to support the cardiologists' 'rule-of thumb' that the pressure drop (in mm Hg) through a stenosis is approximately four times the square of the speed (in m s^{-1}) of the blood jet through the blockage. You may assume that the speed of blood on one side of the stenosis is negligible compared to the speed of blood in the jet.

[7 marks]

1.4) An ultrasound wave of frequency 5 MHz is scattered from red cells in blood moving directly away from the ultrasound source at a speed of $50\,\text{cm s}^{-1}$. What is the wavelength of the ultrasound reflected back to the source? (The speed of sound in the ambient soft tissue, and in blood, is $1500\,\text{m s}^{-1}$.)

[7 marks]

1.5) Use a simple model to show that the rate of decay of the voltage across a membrane is independent of the thickness of the membrane.

[7 marks]

1.6) The time-averaged speed of blood flow is measured to be $20\,\text{cm s}^{-1}$ in the aorta (close to the heart) and $0.5\,\text{mm s}^{-1}$ in the capillaries. The diameter of the aorta is approximately 3200 times that of a typical capillary. Use this information to make a rough estimate of the number of capillaries in the body.

[7 marks]

1.7) Calculate the cardiac output of a patient for whom the following data are known: oxygen consumption, 250 ml/min; venous oxygen content, 0.15 ml/ml; arterial oxygen content, 0.20 ml/ml. (Hint: Use an indicator-dilution method, with oxygen as the indicator.)

[7 marks]

1.8) Comment briefly on the problems to be solved if a surgical implant is to be successful.

[7 marks]

SECTION B: answer TWO questions

2) (a) A renal dialysis arrangement is modelled as a simple two-compartment system. Treat the blood as one compartment, with the dialysis fluid as the second. Make reasonable assumptions to obtain an expression for the time-dependence of the concentration of impurities in the blood. Show that the time-constant describing this behaviour is inversely proportional to the surface area of the dialysis membrane.

[12 marks]

(b) The particle concentration $C(x,t)$ at position x and time t satisfies the diffusion equation, with the following boundary conditions:

$$C(x,t) = C_0 \text{ for } x = 0,\ t \geq 0,$$
$$C(x,t) = 0 \quad \text{for } x > 0,\ t = 0,$$

where C_0 is a constant. Obtain an expression for $C(x,t)$.

Hint: Use Boltzmann's method, introducing a new variable $\xi = \dfrac{x}{\sqrt{4Dt}}$ into the diffusion equation, where D is the diffusion coefficient.

[12 marks]

(c) Indicate how a one-dimensional diffusion equation with a time-dependent diffusion coefficient can be reduced to an equation with a constant diffusion coefficient

[6 marks]

3) (a) Show, with appropriate analysis, how, because the blood flow is pulsatile, the rate of blood flow in a leg may be determined from measurements of changes in the electrical impedance of the limb. Indicate any assumptions made in the calculation

[10 marks]

(b) A linear axon lies along the x-axis. As it depolarises, a constant (dipole) electrical activity vector $\boldsymbol{p}$ sweeps along the cell with constant speed $\boldsymbol{u}$. An electrode at $x = 0$, $y = a$ measures the potential relative to zero at ∞. Obtain an expression for the potential, V, at the electrode as a function of time, t, and sketch it. Assume that at $t = 0$, $\boldsymbol{p}$ is directly under the electrode at $x = 0$.

[12 marks]

(c) Assume that the electrical activity of the heart may be described by a single, dipolar, activity vector. Show how a precordial lead may be used to estimate the component of that vector orthogonal to the plane of the limb-lead components.

[8 marks]

4) (a) Blood flowing in an artery or vein may be modelled as a Newtonian fluid flowing in a rigid tube of constant circular cross section, and driven by a constant pressure gradient. Show that the velocity profile of the flowing blood is parabolic.

[12 marks]

(b) For the model considered in (a), show that the volume flow rate depends on the fourth power of the tube radius. Show also that the spatial average (over the vessel cross section) of the flow speed is half the maximum flow speed in the pipe.

[10 marks]

(c) Show how the area of the aortic valve may be estimated from measurements of the patient's aortic and left-ventricular pressures, cardiac output, ejection period, and heart rate. Be explicit about any approximations made. (Hint: Start from Bernoulli's equation.)

[8 marks]

5) (a) In modelling the propagation of linear pressure waves in tissue, a uniform elastic medium is sometimes assumed. Show that the force law, continuity equation, and constitutive equation for a uniform elastic medium can be combined to give the pressure wave equation:

$$\frac{\partial^2 p}{\partial x^2} - a\frac{\partial^2 p}{\partial t^2} = 0.$$

Write the constant, a, in terms of the density and compressibility of the tissue.

[12 marks]

(b) A more realistic model than that in (a) considers the soft tissue as a uniform Kelvin-Voigt medium. Establish the relationship between pressure and density in such a model. Find the analytical form of the density-pressure creep response in such a tissue.

[10 marks]

(c) Derive Laplace's law for the case of a thin-walled spherical shell. What implications does your result have for the thickness of arterial walls in the body?

[8 marks]