King’s College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/MP33 Medical Engineering

Summer 2003

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A
and TWO questions from SECTION B

The approximate mark for each part of a question is indicated in square brackets

Separate answer books must be used for each Section of the paper

You must not use your own calculator for this paper
Where necessary, a College Calculator will have been supplied

TURN OVER WHEN INSTRUCTED

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SECTION A - Answer SIX parts of this section

1.1) Explain, with the aid of a simple analysis, a demodulation method by which the Doppler shift in a continuous wave Doppler device may be recovered.

[7 marks]

1.2) Laboratory experiments show that the rate of decay of a voltage across a membrane is independent of the thickness of the membrane. Use a simple model to demonstrate that this is an expected result.

[7 marks]

1.3) Derive Laplace's principle for the case of a long thin-walled cylinder. Indicate how this principle leads to an understanding of the mechanical properties of the arterial wall.

[7 marks]

1.4) Calculate the creep response of a simple Kelvin-Voigt medium for which the stress, σ, and strain, ε, are related by $\sigma = a\varepsilon + b\dot{\varepsilon}$, where a and b are constants.

[7 marks]

1.5) The time-averaged speed of blood flow is measured to be approximately $20 cm.s^{-1}$ in the aorta and $0.5 mm.s^{-1}$ in the capillaries. The aortic diameter is approximately $3200 \times$ that of a typical capillary. Use this information to make a rough estimate of the number of capillaries in the body.

[7 marks]

1.6) Explain the indicator-dilution method for measuring blood flow, and show how it can be used to derive an expression for the cardiac output in terms of the arterial and venous concentrations of oxygen, and the rate of oxygen consumption by the body.

[7 marks]

1.7) Show how the blood pressure drop (in mmHg) across a stenosis depends on the speed of the blood (in $m.s^{-1}$) through the blockage. Note that: $1 atm = 10^5$ pascals ≈ 750 mm.Hg.
Hint: Use Bernoulli's theorem.

[7 marks]

1.8) Show how a one-dimensional diffusion equation with a time-dependent diffusion coefficient can be reduced to one with a constant diffusion coefficient.

[7 marks]

SECTION B - Answer TWO questions

2) a) Show how the pulsatile nature of blood flow allows the impedance plethysmography technique to relate measured changes in the electrical impedance of a limb to volume changes in that limb. Indicate any assumptions that are made in the calculation.

[10 marks]

b) An excitation pulse passes along a linear axon, oriented along the x-axis. The depolarisation front of the activated region may be represented by a constant (dipole) electrical activity vector, $\boldsymbol{p}$, sweeping along the cell with constant speed u. An electrode measures the external potential, V, and is located at $x = 0$, $y = a$. Find an expression for V as a function of time, t, and sketch it. Assume that at $t = 0$, $\boldsymbol{p}$ is located directly under the electrode, at $x = 0$.

[10 marks]

c) The electrical activity of the heart may be described by a single, dipolar, activity vector. Show how a precordial lead, when measuring an electrocardiogram, allows an estimate to be made of the component of that vector orthogonal to plane of the limb-lead components.

[10 marks]

3) a) Show how the area of the aortic valve may be estimated from measurements of the patient's aortic and left-ventricular pressures, cardiac output, ejection period, and heart rate.

[8 marks]

b) Blood flow in a vessel is modelled as a viscous Newtonian fluid flowing in a smooth, rigid tube of constant circular cross section, and subjected to a constant pressure gradient. Under these circumstances, the velocity profile of the flowing blood may be estimated to be parabolic. Show that the volume flow rate depends on the fourth power of the radius of the tube, and that the spatial average (over the vessel cross section) of the flow speed is equal to half the maximum blood speed in the tube. Mention three major reasons why this model is unrealistic.

[10 marks]

c) The above model is extended to the case of a blood vessel branching into two identical daughters, but with the simplification that plug flow is assumed. Obtain expressions for the reflection and transmission coefficients of a pressure wave at the junction. It may be assumed that, in this model, the ratio of pressure to speed of flow is equal to $\pm$ (blood density) $\times$(speed of the pressure wave), where the negative sign is used for a backward travelling wave.

[12 marks]

4) a) The mechanical behaviour of a tissue is modelled as a uniform Maxwell medium, which may be represented as a viscous and an elastic element in series.

i) Derive the relationship between stress and strain for such a medium, and interpret this to establish a constitutive equation for the dependence of pressure changes on density perturbations in the model.

[8 marks]

ii) Assume that the only forces which need to be taken into account are scalar pressure forces obeying Newton's second law, that the (mass) continuity equation holds, and that the constitutive equation established above is valid, in order to derive a wave equation for the propagation of pressure waves in such a tissue.

[14 marks]

b) Derive the analytical form of the pressure / density relaxation response for the above tissue model.

[8 marks]

5) a) Starting from first principles, derive the one dimensional continuity equation which links the number flux density to the particle concentration. Use this, and Fick's first law, to derive the diffusion equation (Fick's second law).

[10 marks]

b) Let $C(x,t)$ denote particle concentration at location x at time t. Diffusion of particles takes place across a planar membrane, located at $x = 0$. Initially (at $t = 0$) there are no particles to the right of the membrane ($x > 0$). Also, the particle concentration remains constant at $x = 0$ for all $t \geq 0$. Write the diffusion equation for $\overline{C}(x,s)$, the Laplace transform of C with respect to t, and solve for $\overline{C}$ when $x > 0$.

[10 marks]

c) A renal dialysis machine is modeled as a simple two-compartment system, with the blood and its impurities as one compartment and the dialysis fluid as the second. Make reasonable assumptions to obtain an expression for the time-dependence of the concentration of impurities in the blood, and show that the time-constant describing this process is inversely proportional to the surface area of the dialysis membrane.

[10 marks]