King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CPMP25 Radiation Physics

Summer 2003

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Electronic charge	e	=	1.602×10^{-19} C
Planck constant	h	=	6.626×10^{-34} J s
Speed of light in a vacuum	c	=	2.998×10^{8} m s^{-1}
Rest mass of the electron	m_e	=	9.109×10^{-31} kg
Mass of the proton	m_p	=	1.673×10^{-27} kg
Mass of an α-particle	m_α	=	6.645×10^{-27} kg
Atomic mass unit	m_u	=	1.661×10^{-27} kg

SECTION A - Answer SIX parts of this section

1.1) A calibration lamp has a sharp emission line at 546.07 nm in air. Calculate (to 5 significant figures) the energy of this line in eV and the frequency in Hz. You may take the refractive index of air to be 1.00028.

[7 marks]

1.2) A diatomic molecule has atoms with masses m_1 and m_2. The average separation between the atoms is r. Show that the moment of inertia about the centre of mass is

$$I = \frac{m_1 m_2 r^2}{m_1 + m_2}$$

[7 marks]

1.3) The wavelengths of the copper K_α and copper K_β X-rays are 0.1542 nm and 0.1392 nm, respectively. Stating any assumptions that you make, use this information to estimate the wavelength of the copper L_α line.

[7 marks]

1.4) Describe what is meant by a *population inversion*, and explain why this is necessary for the operation of a laser. Explain briefly, with the aid of a sketch, how population inversion is achieved in a three-level system.

[7 marks]

1.5) Describe *photodynamic therapy* and outline briefly the principles involved.

[7 marks]

1.6) A gamma ray, with energy hn, scattered through an angle q by an electron, gives that electron an energy

$$E = hn\left[\frac{a(1-\cos q)}{1+a(1-\cos q)}\right] \qquad \text{where} \qquad a = \frac{hn}{m_e c^2}.$$

Show that the maximum energy that can be transferred to the electron is

$$\frac{2hn}{(m_e c^2 / hn) + 2}.$$

Calculate this energy for hn = 1.33 MeV.

[7 marks]

1.7) A non-relativistic particle with mass m_1 and kinetic energy Q_1, collides with a stationary particle having mass m_2. It transfers to that particle an energy

$$Q_2 = 4\frac{m_1 m_2}{(m_1 + m_2)^2} Q_1.$$

Show that, for an α-particle colliding with a stationary electron,

$$Q_e \approx \frac{4Q_a m_e}{m_a}.$$

Hence calculate the minimum number of collisions a 4-MeV α-particle must make with electrons in order to reduce its energy to 1.5 MeV. State any assumptions involved.

[7 marks]

1.8) Explain the terms *exposure*, *absorbed dose* and *dose equivalent* used in radiation dosimetry, giving the units in each case.

[7 marks]

SECTION B - Answer TWO questions

2) For a diatomic molecule, treated as a vibrating rotator, the energy levels are given by

$$E_{V,J} = (V + 1/2)\hbar\sqrt{\frac{k}{m}} \quad + \quad \frac{J(J+1)\hbar^2}{2I}$$

where k is the "spring constant", m is the reduced mass of the molecule and I, the moment of inertia, is given by $I = mr^2$ where r is the separation between the atoms. The vibration is assumed to be simple harmonic.

a) Define the terms V and J, and state the selection rules for these parameters.

[2 marks]

b) Illustrate the first few allowed transitions on an energy level diagram. How are energies in Joules converted to the spectroscopic units of cm^{-1}?

[7 marks]

c) Sketch the absorption band for a typical diatomic molecule, and identify the P and R branches. (A derivation of the relative intensities of the transitions is **not** required.)

[5 marks]

d) Show that, near the centre of the band, the spacing of the rotational lines in the spectrum is $2B$, where $B = \frac{\hbar^2}{2I}$.

[4 marks]

e) Chlorine has two stable isotopes, ^{35}Cl and ^{37}Cl. The centre of the fundamental vibration-rotation absorption band for hydrogen chloride $H^{35}Cl$ (i.e. incorporating the light isotope of Cl) is measured as 2885.5 cm^{-1}. Assuming that HCl behaves as a harmonic oscillator, show that, for $H^{37}Cl$ the centre of the band will be shifted approximately 2.17 cm^{-1} to lower energy.

[7 marks]

f) The bond length for HCl is 0.1274 nm. Use this information to calculate the average separation (in cm^{-1}) of the first few peaks in the P or R branch of the absorption spectrum for $H^{35}Cl$.

[5 marks]

3) a) Discuss the three main processes by which gamma rays lose energy on passing through an absorbing medium.

[11 marks]

b) For each case, sketch a curve showing how the probability of the process varies with the energy of the gamma ray.

[9 marks]

c) Describe, in addition, the processes which produce (i) Auger electrons and (ii) gamma rays with an energy of 0.511 MeV.

[6 marks]

d) What thickness of concrete is needed to reduce the number of 500-keV gamma rays in a well-collimated beam to 1 % of the incident number? For concrete the density (ρ) is 2.35×10^3 kg m^{-3} and the mass attenuation coefficient (μ/ρ), for photons of energy 500 keV, is 0.0089 m^2 kg^{-1}.

[4 marks]

4) a) Draw a diagram illustrating the important components in a He/Ne laser, and explain the function of each.

[9 marks]

b) Sketch an energy level diagram which indicates the important transitions leading to laser emission at 632.8 nm.

[4 marks]

c) Explain, with sketches, what is meant by the TEM_{00}, TEM_{01}, and TEM_{10} modes and indicate why the TEM_{00} mode is preferred for many applications.

[5 marks]

d) Explain what is meant by the *gain bandwidth* of the laser. A certain He/Ne laser has mirrors separated by 40 cm. The full width at half maximum of the gain-bandwidth curve is 2×10^{-3} nm. Calculate the number of longitudinal modes in this wavelength interval. Calculate also, in MHz, the frequency spacing of the modes.

[10 marks]

e) In medical applications the He/Ne laser might be used in conjunction with Nd-YAG or CO_2 lasers. Explain why.

[2 marks]

5) a) Explain what is meant by a *survival curve* in the field of radiation therapy. Data for certain groups of cells irradiated with neutrons follow the *single-hit survival curve*, and with X-rays follow the *multi-target single-hit survival curve*. Sketch the forms of these survival curves and derive expressions for the surviving fraction as a function of dose in each case.

[12 marks]

b) Show that, for large doses, the number of cells N surviving after a dose D of X-rays is $N = N_0 \; n \exp(-D/D_0)$, where N_0 is the initial number of cells. Explain the physical significances of the parameters n and D_0.

[4 marks]

c) You may assume that, after irradiation, cell survival follows the Poisson distribution

$$P_m = \frac{a^m e^{-a}}{m\,!}.$$

Define the terms in this expression, and show how it is simplified in the case of no cells surviving.

[4 marks]

d) A large number of batches of cell cultures were irradiated with X-rays and it was found that, on average, a fraction of 10^{-3} survived after a dose of 16.01 Gy and a fraction 10^{-5} survived after a dose of 25.22 Gy. The overall behaviour could be described using the multi-target single-hit survival curve.

The same energy X-rays were used to irradiate a tumour comprising 10^9 similar cells. Calculate the probability of destroying this tumour after a dose of 48 Gy.

[10 marks]