King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

M.Sci. EXAMINATION

CP4750 Image Capture and Sensor Technology

Summer 2004

Time allowed: 3 Hours

Candidates must answer THREE questions.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

You may assume that $\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$

Answer THREE questions

1) With the aid of a suitably labeled diagram, explain the principles of operation of a vacuum photodiode detector, stating the main advantages and disadvantages of this type of detection device.

[5 marks]

A vacuum photodiode is required to register the arrival of a pulse of laser radiation. The electrodes of the vacuum photodiode have an area of 0.5 cm^2 and a separation of 1.0 cm. Calculate the response time of the device, τ, if the voltage applied to it is 300 V.

[4 marks]

Calculate a suitable value for a load resistance R to be used in conjunction with an amplifier, to amplify the output from the above detector, such that the response time of the photodiode is not impaired. Explain the disadvantages of using a smaller value for R.

[4 marks]

The noise equivalent power $(NEP)_{A.L.}$ at the output is now dominated by amplifier noise such that

$$(NEP)_{A.L.} = \frac{2h\nu}{\eta e}\sqrt{\frac{k_B T}{2\tau R}}.$$

Calculate a value for the specific detectivity (D^*) of the detector/amplifier combination, for a wavelength of 1.0 μm, at room temperature and with unit efficiency. Comment on how this would compare to that of an ideal photon detector.

[7 marks]

2) With the aid of a suitably labelled diagram, explain the principles of operation of an avalanche photodiode, stating the main advantages and disadvantages of this type of detection device.

[7 marks]

The electron and hole currents (N, P respectively) in an avalanche photodiode, of width w, may be described by the coupled equations

$$\frac{dN}{dx} = \mu N + \nu P$$

$$\frac{dP}{dx} = -\mu N - \nu P$$

where x is the distance across the semiconductor junction and μ and ν are the field and x-dependent ionisation coefficients for electrons and holes, respectively. Assuming that these ionisation coefficients are equal, show that an avalanche current will occur when

$$\int_0^w \mu \, dx = 1.$$

[6 marks]

An avalanche photodiode, with a full device current gain of Γ and unity Detector Quantum Efficency (DQE), is used to detect an incident illumination of P Watts. Show that for the full device, the mean squared current fluctuation is given by

$$\overline{i_N^2} = 2\frac{P}{h\nu}e^2\Gamma^2 B\left(2 - \frac{1}{\Gamma}\right),$$

where Poisson statistics have been assumed, such that $\overline{i_N^2} = 2eIB$ for a mean signal current I of bandwidth B.

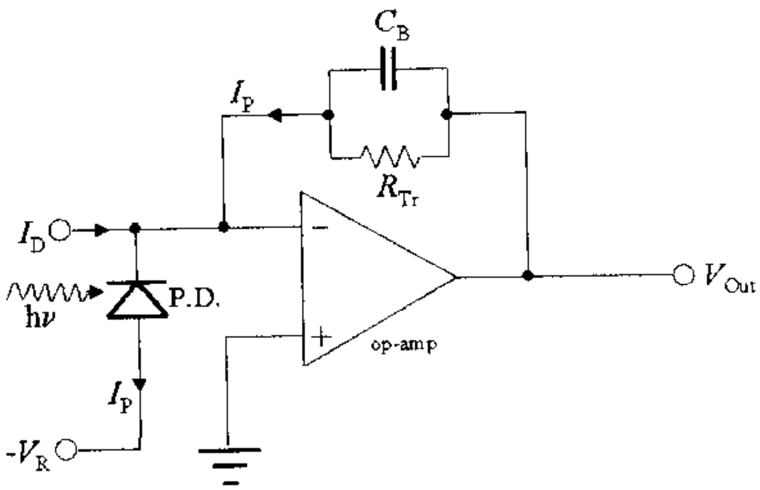
[6 marks]

Comment on how this quantity compares to that of an ideal quantum limited photon detector.

[1 mark]

3) Describe the principles of operation of the following detector circuit, which utilises a photodiode (P.D.) and an *ideal* operational amplifier (op-amp).

[3 marks]



Explain the significance of each of the following elements of the diagram:

a) the supplied voltage $-V_{\mathrm{R}}$,

[1 mark]

b) the supplied current I_{D},

[1 mark]

c) the resistor value R_{Tr}, and

[1 mark]

d) the capacitance value C_{B}.

[1 mark]

Given that the photodiode is illuminated at an intensity of 5 μW cm^{-2}, for which it has a spectral sensitivity of 0.2 Amps W^{-1} cm^{-2}, calculate the voltage developed at V_{Out} if $R_{\mathrm{Tr}} = 1$ MΩ and the area of the photodiode is 1.0 mm $\times$ 1.0 mm.

[3 marks]

Show, explaining the meaning of the symbols used and the asumptions you have made, that the width w of the depletion region at a p-n junction with bias $-V_{\mathrm{R}}$ may be expressed as:

$$w = \sqrt{\frac{2\epsilon_r\epsilon_0\left(-V_{\mathrm{R}} + \Phi\right)}{eN_{\mathrm{A}}\left(1 + \frac{N_{\mathrm{A}}}{N_{\mathrm{D}}}\right)}}.$$

[5 marks]

The p-n junction photodiode has $N_{\mathrm{A}} = 10^{15}$ cm^{-3}, $N_{\mathrm{D}} = 10^{17}$ cm^{-3}, $\Phi = 0.1$ V, and $\epsilon_r = 20$. Estimate the upper limit for the frequency response when the photodiode is operated at a reverse bias of 2.0 V.

[4 marks]

Comment on the need for C_{B}.

[1 mark]

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4) A thermal detector element with total thermal energy E, at a temperature T, has a heat capacity, $C = dE/dT$. Using the Boltzmann relationship for the probability of detector element $p(E_i)$ having an energy E_i is $\propto \exp -(E_i/k_BT)$, show that variance of deviations in temperature is given by:

$$\overline{\Delta T^2} = \frac{k_B T^2}{C}.$$

[7 marks]

The detector has a thermal conductance $G = dP/dT$, where P is the incident thermal power, its frequency response is described by:

$$\overline{\Delta T^2}(\omega) = \frac{A}{1 + \omega^2 \tau^2},$$

where $\tau = C/G$ is the time constant of the device and where $A = 4kT^2/G$ is a constant and is independent of frequency. Show that in a low frequency regime, limited to a bandwidth B, the variance of the temperature deviations is given by:

$$\overline{\Delta T^2} = \frac{4k_B T^2}{G} B.$$

[7 marks]

Hence show that the noise equivalent power (NEP) of such an idealised thermal detector, of unit detection area, is given by:

$$NEP = 4\sqrt{\sigma k_B T^5 B}.$$

[4 marks]

Calculate the specific detectivity (D^*) of the detector when it is at a temperature of $300K$.

[2 marks]

5) Describe blackbody radiation and state Wien's law and Stefan's law.

[3 marks]

Explain what is meant by the terms 'background limited detection' and 'noise equivalent temperature change' ($NE\Delta T$).

[3 marks]

Show that the noise equivalent power, NEP, for a background limited detecting system is given by:

$$NEP = \sqrt{\frac{(2h\nu)(P_{\rm S} + P_{\rm B})B}{\eta}}$$

where the symbols have their usual meanings.

[4 marks]

An object generates temperature fluctuations in a background radiation field received from a scene which has a blackbody temperature of 300 K. An image of this object is formed using a thermal imaging system with an angular field of view of 1 milliradian and an entrance aperture size of $100\,\text{cm}^2$. A cooled filter is incorporated into the imaging system having peak transmittance at $10\,\mu$m with a 10% bandwidth. Given Planck's radiation law

$$dI_\nu = \frac{2h\nu^3 d\nu}{c^2\left(\exp\left(h\nu/k_B T\right) - 1\right)},$$

where the symbols have their usual meanings, calculate a value for $NE\Delta T$ for the imaging system, if the bandwidth required is 1 MHz.

[10 marks]