King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

M.Sci. EXAMINATION

CP4731 The C and C++ Programming Languages

Summer 2004

Time allowed: THREE Hours

Candidates must answer **THREE** questions.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

Physical Constants

Permittivity of free space	ϵ_0	$= 8.854 \times 10^{-12}$	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$= 4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	$= 2.998 \times 10^{8}$	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	$= 6.673 \times 10^{-11}$	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	$= 1.602 \times 10^{-19}$	C
Electron rest mass	m_e	$= 9.109 \times 10^{-31}$	kg
Unified atomic mass unit	m_u	$= 1.661 \times 10^{-27}$	kg
Proton rest mass	m_p	$= 1.673 \times 10^{-27}$	kg
Neutron rest mass	m_n	$= 1.675 \times 10^{-27}$	kg
Planck constant	h	$= 6.626 \times 10^{-34}$	$\mathrm{J\,s}$
Boltzmann constant	k_B	$= 1.381 \times 10^{-23}$	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	$= 5.670 \times 10^{-8}$	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	$= 8.314$	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	$= 6.022 \times 10^{23}$	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		$= 2.241 \times 10^{-2}$	$\mathrm{m^3}$
One standard atmosphere	P_0	$= 1.013 \times 10^{5}$	$\mathrm{N\,m^{-2}}$

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1) Write a program in C which calculates the arcsin ($\sin^{-1}$) of a variable using the following infinite series. Both the argument and the value of the function should be of type 'double'. You should include just enough terms to achieve 5 significant figure accuracy.

$$\arcsin(x) = x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{x^5}{5} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{x^7}{7} + \cdots$$

[18 marks]

Note that $\arccos(x) = \pi/2 - \arcsin(x)$.

Write a further function in C which calculates $\arccos(x)$.

[2 marks]

2) The N real values of the function $f(x)$, sampled at intervals of h between the limits $x = a$ and $x = b$, are stored as formatted values in a file called `data.d` in the order: $N, a, b, f(a), f(a+h), f(a+2h), \cdots, f(b)$ (one 'int' and $2 + N$ 'floats'). Write a program in C which will read this file, allocate memory for the function values, and then differentiate numerically the function $f(x)$. The values of the derivative should be stored in a file with a similar layout, called `differential.d`.

You should use numerical differentiation with a centred difference approach:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

except at the ends of the interval where you will need to use

$$f'(a) = \frac{f(a+h) - f(x)}{h} \quad \text{and} \quad f'(b) = \frac{f(b) - f(b-h)}{h}.$$

[20 marks]

3) Design a class of symmetric matrices in C++. You should include a constructor which allocates memory for the matrix elements (with 2×2 as the default size), a copy constructor, a destructor, overloaded + and =.

[16 marks]

The elements a_{ij} of symmetric tridiagonal matrices are all zero except where $i - 1 \le j \le i + 1$. Include a derived class of tridiagonal matrices. You need only write code for the general constructor and destructor.

[4 marks]

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4) Design a set of functions in C or C++ to handle file input and output simply. That is, provide the same sort of simplification that 'cin' and 'cout' bring to 'scanf' and 'printf', respectively. You will need to use the C functions 'fopen', 'fscanf', 'fprintf' and 'fclose' within your functions. Design them to read or write 'int' variables from or to the files. Illustrate how your functions would be used in a short program.

[15 marks]

Explain how, in C++, this code could be generalised such that other data types could use them. (You do not need to write the code for this part of the question, a description and explanation is adequate.)

[5 marks]

5) Write a function in C or C++ which generates pseudo-random numbers between the limits given as arguments (prototype `float random(float, float);` (You will need to use the C random number generator called `rand()`, which returns a pseudo random 'int' between 0 and `RAND_MAX`; both are in the C 'stdlib' library.)

[5 marks]

The Monte Carlo method of numerical integration (in two dimensions) uses random numbers in the following way. Generate two random numbers which define a point (x, y) within a bigger area which includes the area over which the integration is to take place. If the point is within the area of integration, then calculate the value of the integrand at that point and add it to a cumulative sum; if not, discard that point. Repeat the procedure 10000 times (or more). An estimate of the integral is given by the area of the bigger region times the sum, divided by the number of points generated.

Write a program in C or C++ which uses this method to evaluate

$$\int_D \frac{\exp(-x^2)}{(1+y^4)} dx dy$$

where D is the region defined by $-1 \le x \le 1$, $0 \le y^2 \le (1-x^2)$; that is, a circle of radius 1 centred on the origin in the x, y plane.

[15 marks]