King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP4477 Superconductors, semiconductors and magnetic materials

Summer 2003

Time allowed: THREE Hours

Candidates should answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

permeability of vacuum $\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

electron mass in vacuum $m_o = 9.109 \times 10^{-31}$ kg

effective mass of electron in GaAs $m_e^* = 0.072 m_o$

effective masses of holes in GaAs $m_{hh}^* = 0.6 m_o, \quad m_{lh}^* = 0.15 m_o$

Planck constant $h = 6.626 \times 10^{-34}$ J s^{-1}

electron charge $e = -1.602 \times 10^{-19}$ C

Bohr magneton $\mu_B = 9.274 \times 10^{-24}$ T

Boltzmann constant $k_B = 1.380 \times 10^{-23}$

SECTION B – Answer TWO questions

1)

A two-dimensional square lattice is constructed from identical atoms with one atom on each lattice site. The tight-binding approximation is applied to the non-degenerate atomic orbitals ϕ of the atom at the site (0,0) and its four nearest neighbours at $(\pm a, 0)$ and $(0, \pm a)$. The energy $E(\mathbf{k})$ of an electron with wavevector $\mathbf{k}$ is found to be

$$E(\mathbf{k}) = E_o - \sum_m \beta_m \exp(-i\mathbf{k}.\mathbf{r}_m)$$

where E_o is the atomic energy of the state ϕ and β_m is the matrix element of the electronic Hamiltonian taken between the states at the (0,0) atom and the atom at $\mathbf{r}_m$ respectively.

Show that

$$E(\mathbf{k}) = E_o - 2\beta\left(\cos(k_x a) + \cos(k_y a)\right).$$

[3 marks]

Hence show that

(a) the contours of $E(\mathbf{k})$ form circles in $\mathbf{k}$-space near $\mathbf{k} = 0$,

[3 marks]

(b) when $|k_x| = |k_y| \approx \pi/a$ the contours of $E(\mathbf{k})$ form circles in $\mathbf{k}$-space centred on the four points with $|k_x| = |k_y| = \pi/a$, and

[3 marks]

(c) $E(\mathbf{k})$ is independent of $\mathbf{k}$ when $k_x = (\pi/a) - k_y$.

[3 marks]

Sketch the contours of $E(\mathbf{k})$ in the first Brillouin zone.

[3 marks]

What is meant by the effective mass of an electron? Show that the effective masses of an electron with $\mathbf{k} \approx 0$ and with $k_x \approx k_y \approx \pi/a$ are equal in magnitude but opposite in sign.

[5 marks]

2)

Define the diffusion coefficient D of an impurity in a crystal, in terms of its concentration gradient. Show that, for an interstitial atom, D is related to the distance a between neighbouring stable sites of the impurity by

$$D = \frac{2}{3}\nu a^2 \exp\left(-E_b/kT\right)$$

where ν is the vibrational frequency of the atom and E_b is the energy of the barrier to diffusion.

[6 marks]

By minimising the free energy ($F = U - TS$) of a crystal, show that, in thermal equilibrium, there are $n \approx N \exp(-E/kT)$ interstitial atoms, where N is the number of available sites and E is the energy required to introduce one interstitial atom from the surface of the crystal.

[You may assume Stirling's formula, that $\ln p! \approx p \ln p - p$].

[6 marks]

Describe qualitatively the behaviour of the oxygen atoms in a crystal of silicon when it is grown at 1680 K in an oxygen-rich environment, is then suddenly cooled to room temperature, and is subsequently heated slowly to 720 K ($\approx 450°$ C).

[5 marks]

State briefly why it is often *advantageous* for circuits to be fabricated on silicon which has been grown in an oxygen-rich environment.

[3 marks]

3)

Explain briefly what is meant in the field of semiconductors by the terms

a) epitaxial layer,

b) strained layer,

c) critical thickness,

d) quantum structure, and

e) quantum dot.

[6 marks]

Quantum layer structures are easily made from GaAs and AlGaAs, exploiting the fact that the energy gap of GaAs is smaller than that of AlGaAs. One structure is grown as follows. Starting with a GaAs substrate, a thick AlGaAs layer is grown, followed by a layer of GaAs of thickness x, a 10 nm layer of AlGaAs, a 6.3 nm layer of GaAs and finally a thick capping layer of AlGaAs. The thickness x is unknown, but is less than 6.3 nm.

Sketch the variation in the energies of the edges of the conduction and valence bands as a function of depth in the sample. Where are free holes and free electrons expected to be found?

[5 marks]

The energy E of a particle of mass m trapped in an infinitely deep potential well of width a is $E = h^2n^2/8ma^2$, $n = 1, 2 \ldots$.

State and justify the selection rule for the emission of radiation in a direction perpendicular to the quantum layers when an electron in state n combines with a hole in state n'.

[3 marks]

When the structure is cooled to 5 K and excited with visible light, the lowest energy photons emitted from the two layers have energies of 1.590 and 1.640 eV, respectively. Estimate the effective energy gap of GaAs, and find the value of x.

[6 marks]

4)

(a) Consider a substance with n non-interacting magnetic moments per unit volume, each with allowed magnetic quantum numbers $m_J = \pm\frac{1}{2}$, spin g-factor $g = 2$ and hence magnetic moment $\mu = m_J g \mu_B = \pm\mu_B$. Show that the temperature-dependent part of the volume susceptibility of the spins is given by:

$$\chi = \frac{\mu_o \mu_B n}{B} \tanh\left(\frac{\mu_B B}{k_B T}\right).$$

From this theoretical expression, derive an expression for the Curie constant C of the material.

[8 marks]

(b) If the spins are now considered to interact through an effective field related to the magnetisation, show that in the paramagnetic phase they would obey a law of the Curie-Weiss form:

$$\chi = \frac{C}{T - \theta_c} \quad \text{for} \quad T > \theta_c$$

with the same Curie constant as in (a).

After correction for core diamagnetism, a sample of a hypothetical metal is observed to have a low-field volume magnetic susceptibility which can be well described by:

$$\chi = \frac{M}{H} = \frac{C}{T - \theta_c} + \alpha, \quad \text{for} \quad T > \theta_c.$$

The experimental values found are $\theta_c = 100$ K, $C = 0.313$ K and $\alpha = 3.7 \times 10^{-7}$.

Confirm that the experimental value of C is consistent with a number density of spins $n = 4 \times 10^{28}$ m^{-3}. Estimate the mean field parameter, and the value of the exchange field (expressed in tesla) at 120 K in an applied field of 0.1 T. Describe briefly the physical origin of this exchange field.

[5 marks]

(c) The Pauli susceptibility of an electron gas is given by,

$$\chi = \mu_o \mu_B^2 g(E_F)$$

where $g(E_F)$ is the density of electronic states at the Fermi energy. What value of $g(E_F)$ is predicted by the temperature-independent part of χ of the hypothetical metal described in (b)? Explain briefly why the Pauli susceptibility is so much smaller than the Curie susceptibility.

[4 marks]

(d) As the temperature is lowered below the Curie temperature in zero applied field, the magnitude of the spontaneous magnetisation observed by neutron diffraction experiments increases. However the bulk magnetic moment of the sample remains zero. By considering the measurement techniques involved, reconcile these observations.

[3 marks]

5)

Describe, briefly, the following properties of a type I superconductor:

i) the temperature dependence of its resistance,

ii) the effect of a magnetic field on the critical temperature,

iii) the variation of specific heat with temperature near the critical temperature,

iv) the effect of isotopic substitution on the critical temperature,

v) the variation of transmission for infra-red radiation.

[10 marks]

The current density **j** associated with a Cooper pair can be written in terms of the order parameter ψ and the magnetic vector potential **A** as

$$\mathbf{j}(\mathbf{r}) = \frac{i\hbar e}{2m}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) - \frac{2e^2}{m}\psi^* \psi \mathbf{A}.$$

Show that the current density through a Josephson 'weak link' is $j = j_o \sin \delta$, where j_o is a constant and δ is the phase difference of the order parameter between the two sides of the link.

[3 marks]

A constant voltage V is applied across the junction. Show that the current density is

$$j = j_o \sin\left(\frac{2eV}{\hbar}t + \delta_o\right)$$

where δ_o is a constant.

[4 marks]

Describe briefly how this weak link can be used to determine the ratio h/e.

[3 marks]