Speed of light $c = 2.998\times10^{8}\,\mathrm{m\,s^{-1}}$

Boltzmann constant $k = 1.381\times10^{-23}\,\mathrm{J\,K^{-1}}$

Planck constant $h = 6.626\times10^{-34}\,\mathrm{J\,s}$

Reduced Planck constant $h = h/2\pi$

Proton mass $m_p = 1.673\times10^{-27}\,\mathrm{kg}$

Electron mass $m_e = 9.109\times10^{-31}\,\mathrm{kg}$

Atomic number of carbon $= 6$

Atomic number of oxygen $= 8$

Atomic weight of oxygen ≈ 16

Electron charge $e = -1.602\times10^{-19}\,\mathrm{C}$

Electron volt, $1\,\mathrm{eV} = 1.602\times10^{-19}\,\mathrm{J}$

Permittivity of vacuum $e_0 = 8.854\times10^{-12}\,\mathrm{F\,m^{-1}}$

Permeability of vacuum $m_0 = 4\pi\times10^{-7}\,\mathrm{N\,A^{-2}}$

Ionisation potential of neutral hydrogen $= 13.6\,\mathrm{eV}$

SECTION A – Answer SIX parts of this section

1.1) Place the following examples of plasmas in order of (a) increasing temperature and (b) increasing electron density: (i) the Solar wind,(ii) a laser-generated plasma for use as a source of soft x-rays, (iii) the ionosphere. Name one other plasma which can be hotter and denser than any of those listed.

[7 marks]

1.2) A plasma consisting of ionised oxygen has an electron temperature of 50 eV. Assuming that the electron and ion temperatures are equal, estimate the mean electron and ion speeds.

[7 marks]

1.3) Obtain an expression for the rise in electric potential in a small spherical region of a singly ionised plasma in which the local electron and ion densities, n_e and n_i, are unequal. Hence define the *Debye Length* l_D and show that it is given approximately by the expression $l_D = (3e_0kT/e^2n_e)^{1/2}$ at temperature T K.

[7 marks]

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1.4) *Local thermodynamic equilibrium* is used to describe plasmas with electron densities n_e larger than about $10^{19} \times c^3 \sqrt{T_{eV}}\ m^{-3}$, where T_{eV} is the electron temperature in electron volts and c is the highest ionisation potential, in electron volts, of any ion present in the plasma. State one condition which must be satisfied for this expression to be true. Estimate the lower limit on n_e for a hydrogenic carbon plasma with $T_{eV} = 80$ eV, and state which form of equilibrium will become appropriate as the electron density decreases slightly.

[7 marks]

1.5) The intensity $I(l)$ as a function of wavelength l for a Doppler broadened line with a central wavelength l_0 is given by the Gaussian profile

$$I(l) = I(l_0)\exp\left[-\frac{(l_0 - l)^2 c^2}{2v_{ion}^2\, l_0^2}\right].$$

where v_{ion} is the mean speed of the ions along the line of sight. Derive an expression for the full width at half maximum (FWHM) of this profile. The FWHM of a Doppler broadened line with $l_0 = 1.9$ nm in an oxygen plasma is measured to be 0.1 pm. What is the ion temperature of the plasma?

[7 marks]

1.6) A plasma is formed by focusing a pulsed laser beam of energy 50 mJ and pulse length 0.5 ns to a spot diameter of 20 µm on a low atomic number material. Determine the *irradiance*. The soft x-ray emission from the plasma consists primarily of lines from helium-like ions. Describe qualitatively what happens to the spectrum as the laser beam energy is gradually increased.

[7 marks]

1.7) Briefly discuss the problems that had to be overcome in order to realise plasma-based soft x-ray lasers.

[7 marks]

1.8) A plasma consisting mainly of hydrogenic carbon ions has a mean electron speed of $\approx 7\times10^6\ ms^{-1}$ and an electron density of $\approx 10^{30}\ m^{-3}$. Although most of the soft x-ray emission of such a plasma will be in discrete spectral lines, there will also be some bremsstrahlung emission. Show that quantum mechanical effects must be taken into account in the theoretical description of the bremsstrahlung.

[7 marks]

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SECTION B – Answer TWO questions

2) a) What causes spectral lines to be naturally broadened? Why is it not normally possible to observe such broadening in the line emission from plasmas?

[3 marks]

b) Show that natural broadening results in a Lorentzian profile, $I(n) = I(n_0)/(1+A^2 t^2)$. Define the parameters n_0 and t and derive an expression for A.

[The Fourier transform of a function $f(t)$ is $G(n) \propto \int_{-\infty}^{\infty} f(t)\exp(-2\text{pi}n\, t)\mathrm{d}t$]

[8 marks]

c) A spectral line at a wavelength $l = 4.02\,\text{nm}$, in a plasma consisting of helium-like carbon, results from a transition from an upper ionic level with a lifetime of 10^{-12} s to the ground state. Using the result of part b), or otherwise, determine the natural width Δl of this line.

[8 marks]

d) Analysis of the measured spectrum of a similar plasma shows that the profile of the 4.02 nm line is approximately Lorentzian with a width $\Delta l \approx 10\,\text{pm}$. Which process is likely to be the dominant one causing this line shape and width? Estimate a characteristic time associated with this process. Discuss qualitatively the properties of a plasma in which such broadening is likely to be observed.

[8 marks]

e) State, with a brief explanation, one other effect which is likely to contribute to the line profile in the plasma of part d). You may assume that the plasma temperature is quite low.

[3 marks]

3) a) Why is the fusion of deuterium with tritium the favoured process for a fusion reactor?

[3 marks]

b) Briefly describe two forms of pre-ignition heating which can be used in a fusion reactor.

[6 marks]

c) Describe qualitatively, and with the aid of a diagram, the magnetic field structure of a tokamak.

[6 marks]

d) Show how magnetic fields can be used to trap charged particles and how this relates to plasma confinement in a tokamak. Distinguish between trapped and passing electrons and discuss the effects that the latter may have.

[12 marks]

e) At the weakest field point, $B_0 = 1\text{T}$ in a particular tokamak, the velocity component along **B** is twice that in the transverse direction. Determine the value of B at which the particles are trapped.

[3 marks]

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4) a) Describe, with the aid of a suitably labelled energy-level diagram, the processes of *bremsstrahlung* and *recombination radiation*. What is meant by *classical bremsstrahlung*?

[6 marks]

b) The power spectrum for classical bremsstrahlung from an electron of initial speed v may be written as

$$\frac{\mathrm{d}P}{\mathrm{d}w} = \frac{Z^2 e^6}{(4\mathrm{p}e_0)^3} \frac{16 n_\mathrm{i}}{3 m_\mathrm{e}^2 c^3 v} \frac{\mathrm{p}}{\sqrt{3}} G$$

where n_i is the ion density and the other symbols have their usual meanings. Explain the significance of the *Gaunt factor*, G.

[4 marks]

c) Modify the above equation for the bremsstrahlung power spectrum to give an expression for the power emitted in recombination radiation from an electron of initial speed v in a hydrogenic plasma, explaining your reasoning. You may assume that for hydrogenic ions of charge Ze the energy of a bound state with principal quantum number n is given by

$$E_n = -\frac{Z^2 e^4 m_\mathrm{e}}{2(4\mathrm{p}e_0\hbar)^2} \frac{1}{n^2}.$$

[10 marks]

d) For a Maxwellian distribution of electron speeds the ratio of powers emitted in recombination radiation (RR) and bremsstrahlung (B) is given by

$$\frac{\mathrm{RR}}{\mathrm{B}} = -\frac{2E_n}{nT_\mathrm{eV}} \frac{G_n}{\bar{g}(w, T_\mathrm{eV})} \exp\left(-\frac{E_n}{T_\mathrm{eV}}\right)$$

where E_n and T_eV (the electron temperature) are in electron volts. Briefly discuss the two Gaunt factors, G_n and $\bar{g}(w, T_\mathrm{eV})$. Use this equation to plot the ratio of intensities in recombination radiation and bremsstrahlung for a plasma consisting of hydrogenic oxygen ions and for electron temperatures of 50 eV and 500 eV and principal quantum numbers of 1, 4 and 10, explaining any assumptions. Discuss your results.

[10 marks]

SEE NEXT PAGE

5) a) In the fluid model used for describing plasma dynamics, the momentum equation is

$$m_j\left(\frac{\partial}{\partial t}+\vec{\mathbf{v}}_j.\nabla\right)\vec{\mathbf{v}}_j=-\frac{1}{n_j}\nabla P_j+q_j\left(\vec{\mathbf{E}}+\vec{\mathbf{v}}_j\times\vec{\mathbf{B}}\right).$$

Define each of the quantities dependent upon j in this equation. Briefly explain how the equation is obtained from the Vlasov equation.

[6 marks]

b) Starting from the momentum equation, derive the following two magnetohydrodynamic equations,

$$r\frac{\partial\vec{\mathbf{v}}_j}{\partial t}=\vec{\mathbf{j}}_j\times\vec{\mathbf{B}}_0$$

$$\vec{\mathbf{E}}+\vec{\mathbf{v}}_j\times\vec{\mathbf{B}}_0=0$$

stating any approximations. Define the quantities $\vec{\mathbf{j}}_j$ and $\vec{\mathbf{B}}_0$.

[9 marks]

c) Show that for a low density plasma a solution of these magnetohydrodynamic equations is

$$\mathrm{i}m_0 j_x=\frac{wE_x}{v_A^2},\quad \mathrm{i}m_0 j_y=\frac{wE_y}{v_A^2}$$

stating any assumptions. Hence obtain an expression for the Alfvén speed v_A, which you should check is dimensionally correct. Estimate the Alfvén speed at the base of the solar corona where the electron density is $\approx 4\times10^{14}\,\mathrm{m}^{-3}$ and the magnetic flux density is $\approx 10^{-4}\,\mathrm{T}$.

[The derived SI units Tesla and Newton are, respectively, $\mathrm{kg\,s^{-2}\,A^{-1}}$ and $\mathrm{m\,kg\,s^{-2}}$.]

[15 marks]

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