# King's College London

UNIVERSITY OF LONDON



B.Sc. EXAMINATION

CP/3650 Introductory plasma physics

Summer 2000

Time allowed: THREE Hours

**Candidates must answer SIX parts of SECTION A,**
**and TWO questions from SECTION B.**

**Separate answer books must be used for each Section of the paper.**

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

**TURN OVER WHEN INSTRUCTED**

Speed of light $c$ = $2.998\times10^{8}\,\mathrm{m\,s^{-1}}$

Boltzmann constant $k$ = $1.381\times10^{-23}\,\mathrm{J\,K^{-1}}$

Proton mass $m_\mathrm{p}$ = $1.763\times10^{-27}\,\mathrm{kg}$

Electron mass $m_\mathrm{e}$ = $9.109\times10^{-31}\,\mathrm{kg}$

Electron charge $e$ = $-1.602\times10^{-19}\,\mathrm{C}$

Electron volt, 1 eV = $1.602\times10^{-19}\,\mathrm{J}$

Permittivity of vacuum $\varepsilon_0$ = $8.854\times10^{-12}\,\mathrm{F\,m^{-1}}$

Atomic weight of iron $\approx 56$

## SECTION A – Answer SIX parts of this section

1.1) Present an argument which shows that a typical plasma cannot be described in an exact way.

[7 marks]

1.2) Give three examples of astrophysical plasmas, two examples of naturally occurring terrestrial plasmas and two examples of man-made plasmas.

[7 marks]

1.3) Define the *electron plasma temperature*, $T$. If $T = 10^{6}\,\mathrm{K}$ determine the corresponding value $T_\mathrm{eV}$ in electron volts.

[7 marks]

1.4) Show that, over appropriate length and time scales, a typical plasma is electrically neutral.

[7 marks]

1.5) A charge $q$ in a plasma moves under a potential $V$ described by the equation

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}V}{\mathrm{d}r}\right)\approx\frac{2e^2n_\mathrm{e}}{\varepsilon_0 kT}V$$

where the symbols have their usual meanings. Show, by substitution, that a suitable form for $V$ is $(q/4\pi\varepsilon_0 r)\exp(-r/\lambda_\mathrm{D})$. Hence find an expression for the *Debye length* $\lambda_\mathrm{D}$.

[7 marks]

1.6) Describe, with the aid of a suitably labelled energy-level diagram, the three main processes by which a plasma emits radiation. Write down two other plasma processes via which radiation could be emitted, and state why these are not normally significant.

[7 marks]

1.7) Under what conditions is *coronal equilibrium* suitable for describing a plasma? What determines the populations of the ionic energy levels in such a plasma? Discuss the transition between coronal equilibrium and local thermodynamic equilibrium.

[7 marks]

1.8) Describe how a plasma that would be suitable for an x-ray laser could be formed.

[7 marks]

## SECTION B – Answer TWO questions

2) a) Define what is meant by the *plasma frequency* $\nu_P$.

[3 marks]

b) Show, explaining each step, that the equation of motion for a displaced electron in a plasma, assuming that thermal energy and collisions can be neglected, is

$$m_e \ddot{x} + \left(e^2 n_e / \varepsilon_0\right) x = 0,$$

where the symbols have their usual meanings. Hence derive an expression for $\nu_P$ and calculate the value of $\nu_P$ for a plasma with electron density $10^{24}\,\mathrm{m}^{-3}$.

[13 marks]

c) By considering the behaviour of the free electrons in a plasma under the influence of an external electromagnetic field, derive an expression for the plasma refractive index $n_P$.

[12 marks]

d) Calculate the frequency below which an electromagnetic wave cannot propagate in a plasma with electron density $10^{24}\,\mathrm{m}^{-3}$.

[2 marks]

3) a) What is the *principle of detailed balance*?

[2 marks]

b) Define the *Einstein coefficients*. Use the principle of detailed balance and the blackbody formula

$$w_\omega = \frac{n_P^2 \hbar \omega^3}{2\pi^2 c^3} \frac{1}{e^{\hbar\omega/kT} - 1},$$

where $w_\omega$ is the spectral energy density and $n_P$ is the plasma refractive index, to derive relationships between the Einstein coefficients.

[12 marks]

c) If two energy levels $E_1$ and $E_2$ have statistical weights $g_1 = 2$ and $g_2 = 6$ determine the ratio of number densities $n_2/n_1$ above which amplification of an incident beam of radiation can occur. What other condition must the incident beam satisfy in order for amplification to occur?

[6 marks]

d) Describe the effects which contribute to the line profile of transitions between the energy levels $E_1$ and $E_2$. State how the line profile can give information about the plasma.

[10 marks]

4) a) Describe the origin of the *solar wind* and how it creates a plasma environment in the vicinity of the Earth.

[10 marks]

b) Draw a diagram showing the plasma structure of the Earth's magnetosphere. Show on the diagram typical electron densities, electron temperatures and magnetic field strengths for each of the magnetosphere regions.

[10 marks]

c) Describe the origin of the *ionosphere* and the *aurora*.

[5 marks]

d) Imagine that the Earth is moved slightly further from the Sun. Discuss the effect this would have on the ionosphere and the aurora.

[5 marks]

5) a) What processes take place in the formation of a *laser produced plasma*?

[8 marks]

b) A pulsed laser beam of energy 100 mJ and pulse length 1 ns is focused to a spot diameter of 10 μm. Determine the *irradiance*.

[4 marks]

c) Describe the x-ray emission spectrum that would result from the formation of a plasma from a low *Z* target. Discuss, qualitatively, how the spectrum would vary with increasing irradiance. Assuming that the irradiance is kept constant, what is the effect on the x-ray emission that increasing the pulse length might have?

[9 marks]

d) Describe briefly how hydrodynamic models can be used to predict the emission characteristics of a plasma. Discuss briefly the advantages and disadvantages of hydrodynamic models compared with particle in cell models.

[9 marks]