King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP3402 Solid State Physics

January 2005

Time allowed: THREE Hours

Candidates should answer all SIX parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ε_0	=	8.854×10^{-12}	F m^{-1}
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Speed of light in free space	c	=	2.998×10^{8}	m s^{-1}
Gravitational constant	G	=	6.673×10^{-11}	N m^2 kg^{-2}
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
Boltzmann constant	k_B	=	1.381×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	W m^2 K^{-4}
Gas constant	R	=	8.314	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.022×10^{23}	mol^{-1}
Molar volume of ideal gas at STP		=	2.241×10^{-2}	m^3
One standard atmosphere	P_0	=	1.013×10^{5}	N m^{-2}

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SECTION A – Answer all SIX parts of this section

1.1 Describe the *caesium chloride* (CsCl) crystal structure.
The ionic radii of Cs^+ and Cl^- are 0.167 nm and 0.181 nm, respectively. Use this information to calculate the lattice constant of CsCl.

[7 marks]

1.2 Explain how the *Miller Indices* are calculated for a plane of atoms in a crystal, and write down a general expression for the separation of planes in a cubic crystal.
A plane in a cubic crystal makes intercepts of 1, 1, and 2 units on the x, y and z axes, respectively. Show that the separation of planes with this orientation is $a/3$ where a is the lattice constant.

[7 marks]

1.3 Sketch the heat capacity as a function of temperature T, for a non-metallic crystal, over the range $T = 0$ to $\sim 2\theta_D$, where θ_D is the Debye temperature.
Calculate an approximate value for the heat capacity of 0.1 kg of germanium at high temperature. The atomic mass number of Ge is 74.

[7 marks]

1.4 Define the terms in the expression $E_y = R_H J_x B_z$ for the *Hall Effect* with orthogonal geometry.
A rectangular block of an n-type semiconductor, with an electron concentration of 10^{19} m^{-3}, has dimensions x = 10 mm, y = 2 mm and z = 1 mm. Calculate the Hall voltage when a current of 2 mA flows along the x direction, and the specimen is placed in a magnetic flux density of 0.5 T along the z direction.

[7 marks]

1.5 Using sketches where appropriate,
(a) Explain what is meant by an *indirect gap* semiconductor, and
(b) Describe two processes by which electrons may be excited optically from the top of the valence band to the bottom of the conduction band in such a material.

[7 marks]

1.6 Explain:
(a) what is meant by a *depletion layer* in a p-n junction,
(b) why a p-n junction possesses electrical capacitance.

[7 marks]

SECTION B – Answer TWO questions

2. (a) Explain the meanings of the terms in the expression for the structure factor F_{hkl} in relation to the diffraction of X-rays from a crystalline solid:

$$F_{\mathrm{hkl}} = \sum_j f_j \exp\left[2\pi i\,(h\,x_j + k\,y_j + l\,z_j)\right].$$

[5 marks]

(b) Prove that X-ray diffraction from crystals with the face centred cubic (FCC) structure requires that h, k and l must be all odd or all even.

[8 marks]

(c) Show that, for a cubic crystal, the Bragg law may be written in the form $\sin^2\theta = \dfrac{\lambda^2}{4a^2}N$, where $N = (h^2 + k^2 + l^2)$ and the symbols have their usual meanings.

[2 marks]

(d) X-ray diffraction was measured from a polycrystalline crystal, known to have the FCC structure, using X-rays with wavelength 0.1600 nm. The equipment allowed only a limited range of angles to be covered. Two *adjacent* diffraction signals were observed at angles of 124.86° and 150.02°, measured with respect to the direction of the incoming X-ray beam. By drawing up a table of allowed values of N, use this information to calculate the lattice constant of the crystal.

[11 marks]

(e) Explain why the technique in (d) could yield an incorrect value for the lattice constant.

[4 marks]

3. (a) Explain the relevance of the Fermi energy at a temperature of 0 K for a metal crystal.

[2 marks]

(b) Starting from the Schrödinger equation, or otherwise, show that the Fermi energy for N free electrons in a metal crystal of volume V is given by

$$E_{\mathrm{F}} = \frac{h^2}{8m_{\mathrm{e}}}\left(\frac{3N}{\pi V}\right)^{2/3}$$

[13 marks]

(c) Calculate the Fermi energy in eV for aluminium (Al), which is trivalent. The atomic mass number of Al is 27 and the density is 2700 kg m^{-3}.

[7 marks]

(d) The heat capacity for free electrons in a metal is given approximately by $C_{\mathrm{V}} = \frac{\pi^2}{2} N k_{\mathrm{B}}\left(\frac{T}{T_{\mathrm{F}}}\right)$, where T_{F} is the Fermi temperature. Calculate C_{V} for Al at a temperature of 290 K, and explain why it differs considerably from the classical value.

[8 marks]

4. (a) Explain, with an example, what is meant by a *substitutional donor*.

[4 marks]

(b) The ionisation energy E_1, and the first Bohr radius r_1 for the hydrogen atom are given by

$$E_1 = \frac{m_e e^4}{2\hbar^2 (4\pi\varepsilon_0)^2} = 13.6 \text{ eV} \quad \text{and} \quad r_1 = \frac{\hbar^2}{m_e e^2} 4\pi\varepsilon_0 = 0.053 \text{ nm} .$$

Show how these equations may be modified to estimate the ionisation energies and Bohr radii of substitutional group V donors in a group IV semiconductor.

[8 marks]

(c) The effective mass of electrons in silicon is 0.33 m_e and the relative permittivity of the bulk material is 11.7. Use these data to calculate approximate values for the donor ionisation energy and first Bohr radius of a group V donor.

[4 marks]

(d) Explain how features in the absorption spectrum associated with such a donor may be interpreted in a simplistic way, using the hydrogen model analogy.

[7 marks]

(e) The lattice constant of the cubic unit cell for silicon is 0.5431 nm. Using the results from (c), calculate the approximate number of silicon atoms that are within one Bohr radius of a group V donor.

[7 marks]