King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/2380 Electromagnetism

Summer 2002

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

Permittivity of free space $\varepsilon_0 = 8.854\times10^{-12}$ Fm^{-1}
Permeability of free space $\mu_0 = 4\pi\times10^{-7}$ Hm^{-1}

Section A – Answer SIX parts of this section.

1.1) Derive the expression for the external work done in charging a capacitor of capacitance C by a source that produces a potential difference V.

[7 marks]

1.2) State the integral form of Gauss's Law as it applies in electrostatics, illustrating your answer with an appropriate diagram. Starting with this integral equation, show that the divergence of the electric displacement **D** is equal to the free charge density ρ.

[7 marks]

1.3) Give a brief account of the phenomenon of paramagnetism.

[7 marks]

1.4) The Cartesian components of the electric field strength **E** of a particular electromagnetic field are given as

$$E_x = E_{x0}\exp\left[i(wt - kz)\right] \qquad E_y = E_{y0}\exp\left[i(wt - kz)\right]\exp\left[id\right] \qquad E_z = 0$$

Describe the various *polarisation states* of this wave field. Your answer should indicate the influence of the wave amplitudes E_{x0} and E_{y0} and the phase angle δ on the polarisation state.

[7 marks]

1.5) When a dielectric substance is subjected to an electric field of strength **E** it becomes *polarised*. Discuss the meaning of the term *polarised*. Define the polarisation **P** and electrical susceptibility χ of the dielectric.

[7 marks]

1.6) Show that the quantity ½**E.D** is an expression for the energy density (U) of an electrostatic field, where **E** is the electric field vector and **D** the electric displacement. What is the energy density associated with an electric field of 10^6 Vm^{-1} in free space?

[7 marks]

1.7) Derive the boundary conditions for the magnetic field **H** and the magnetic flux density **B** at the interface between two homogeneous and isotropic media.

[7 marks]

1.8) In a region of empty space devoid of electric charge or current, show that the magnetic flux density **B** may be derived from a scalar potential f satisfying Laplace's equation and such that $\mathbf{B} = -m_0\nabla f$.

[7 marks]

Section B – Answer TWO questions from this section.

2. Give a definition of the electric field strength **E**.

[2 marks]

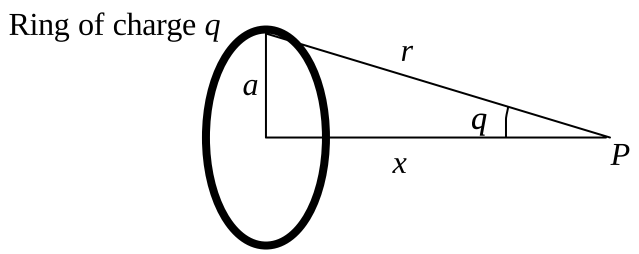
Electric charge is distributed through a free-space volume V with a volume charge density ρ. Use the Coulomb law of force between two electrical charges to derive a formula for the electric field strength produced by this volume distribution.

[6 marks]

Show that the magnitude of the electric field strength $|\mathbf{E}|$ at a point P on the axis of a uniform ring of charge q and radius a is given by the expression

$$|\mathbf{E}| = \frac{q\cos\theta}{4\pi\epsilon_0 r^2},$$

where r and θ are defined in the following diagram.



[8 marks]

Write $|\mathbf{E}|$ as a function of x and show that the maximum values of $|\mathbf{E}|$ occur at a distance $x = \pm a/\sqrt{2}$.

[4 marks]

Sketch a graph showing the electric field strength as a function of x.

[4 marks]

If an electron is placed at the centre of the ring and then is displaced a small distance along the x-axis ($x \ll a$), show that it oscillates with a frequency

$$\nu = \sqrt{\frac{eq}{16\pi^3\epsilon_0 a^3 m}},$$

where m is the mass of the electron.

[6 marks]

3. State the integral form of Ampère’s Law that relates the magnetic field strength $\mathbf{H}$ to the current I flowing in a conductor and explain all the terms used.

[3 marks]

A long, straight wire is situated in free space. The axis of the wire defines the z-direction of the cylindrical polar coordinate system (r, f, z), with $\hat{\mathbf{f}}$ and $\hat{\mathbf{z}}$ representing unit vectors in the f and z directions, respectively. The wire is of radius a = 1 mm and the relative permeability μ_r = 1001; it carries a current I = 10 A which is uniformly distributed over the cross section of the wire.

Derive equations for the cylindrical polar components of the field strength $\mathbf{H}$ and the magnetic flux density $\mathbf{B}$ for $r \geq a$.

[5 marks]

and for $r \leq a$.

[5 marks]

Sketch the variation of $\mathbf{H}$ and $\mathbf{B}$ with r, for $0 < r < 10$ mm

[5 marks]

Show that the magnetisation $\mathbf{M}$ of the substance of the wire is given by the formula

$$\mathbf{M} = \hat{\mathbf{f}}\,\frac{(m_r - 1)rI}{2p\,a^2}, \qquad r \leq a$$

[6 marks]

Assuming the relation between the Ampèrian volume current density and curl $\mathbf{M}$, derive a formula for the volume Ampèrian current density and show that the numerical value of the total Ampèrian current inside the wire is 10,000 A.

[It is given that $\nabla \times \mathbf{M} = \hat{\mathbf{z}}\left[\frac{1}{r}\frac{\partial (rM_f)}{\partial r}\right]$]

[6 marks]

4. In electromagnetism, a magnetic vector potential **A** may be defined so that the magnetic induction **B** is given as $\mathbf{B} = \nabla \times \mathbf{A}$. Why is it possible to express **B** in this way?

[3 marks]

Use Maxwell's curl **E** equation to prove that the electric field strength **E** can be expressed as

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

[5 marks]

A z-directed current dipole located at the origin of a Cartesian co-ordinate system in free space produces a magnetic vector potential **A**=(0,0,A_z). **A** must satisfy the Helmholtz equation which may be written as

$$\frac{1}{r}\frac{\partial^2 (rA_z)}{\partial r^2} + k^2 A_z = 0 \quad ,$$

where $k^2 = w^2 me$. Show that a solution for A_z is

$$A_z = \frac{a}{r}\exp\left[i(wt - kr)\right]$$

where a is a constant.

[4 marks]

Use A_z to determine A_q and A_r in the spherical polar co-ordinate system, and hence show that in spherical polar coordinates, the f-component of **B** is given by

$$B_f = a\left(ik + \frac{1}{r}\right)\frac{\exp\left[i(wt - kr)\right]}{r}\sin q$$

[You may use the formula: $(\nabla \times \mathbf{A})_f = \frac{1}{r}\left(\frac{\partial (rA_q)}{\partial r} - \frac{\partial A_r}{\partial q}\right)$]

[6 marks]

Comment on the form of B_f and determine an approximate expression for the f-component of the magnetic field strength H_f in the *radiation zone*.

[5 marks]

Use Maxwell's $\nabla \times \mathbf{H}$ equation and your expression for H_f in the *radiation zone* to show that

$$E_q = \frac{iak^2 \exp\left[i(wt - kr)\right]\sin q}{we_0 m_0 r}$$

[You may use the formula: $(\nabla \times \mathbf{H})_q = -\frac{1}{r}\frac{\partial (rH_f)}{\partial r}$]

[5 marks]

How does the power flow away from the dipole vary with direction in the *radiation zone*?

[2 marks]

5. The electric field **E** in a homogeneous, linear, isotropic medium with conductivity s, relative permittivity e_r and relative permeability m_r, satisfies the equation

$$\nabla^2\mathbf{E} - s\, m_0 m_r \frac{\partial \mathbf{E}}{\partial t} - e_0 m_0 e_r m_r \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

Starting from the appropriate Maxwell's equations derive the above equation and explain how the second term in the equation arises.
[You may use the formula: $\nabla\times(\nabla\times\mathbf{E}) = \nabla(\nabla.\mathbf{E}) - \nabla^2\mathbf{E}$]

[8 marks]

By considering a plane wave propagating in the z-direction with $E_x = E_0 \exp[i(wt - kz)]$ and $E_y = 0$, where E_0 is a constant and all other symbols have their usual meaning, show that, the medium described above supports propagation of electromagnetic waves, but with amplitude attenuation.

[8 marks]

For a good conductor, show that the attenuation length δ may be approximated by

$$\sqrt{\frac{2}{s w m_0 m_r}}.$$

Explain clearly the assumptions you make in deriving this answer.

[8 marks]

For an electrically screened laboratory it is desired that electromagnetic radiation of frequency 10^6 Hz should be attenuated by at least 100dB. Screening will be provided by lining the walls with thin copper sheeting. What is the minimum thickness of copper required?

[The conductivity of copper is 5.8×10^7 $(\Omega\mathrm{m})^{-1}$ and take m_r=1. An amplitude A is n dB (decibels) weaker than amplitude B if $20\log_{10}(B/A) = n$.]

[6 marks]