King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP1480 Fields and Waves

January 2006

Time allowed: THREE Hours

Candidates should answer ALL parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$
Mass of the Earth	M_E	=	5.97×10^{24}	kg
Radius of the Earth	R_E	=	6.380×10^{6}	m
Mass of the Sun	M_S	=	1.99×10^{30}	kg
Radius of Earth–Sun orbit		=	1.50×10^{8}	km

The sides of a triangle are related by $c^2 = a^2 + b^2 - 2ab\cos(a,b)$.

$$\sin A + \sin B = 2\cos\left[\tfrac{1}{2}(A-B)\right]\sin\left[\tfrac{1}{2}(A+B)\right].$$

SECTION A – Answer ALL parts of this section

1.1) A thin lens of focal length 5 cm is used to produce a focused image of a house on a screen. The house has a height of 10 m and is 50 m from the lens. How far must the screen be from the lens? Calculate the height of the image, and state whether it will be inverted or the same way up as the house.

[7 marks]

1.2) Flint glass has a refractive index of 1.66 for violet light, and 1.61 for red light. A narrow beam of white light inside a block of the glass approaches the plane boundary between the glass block and a vacuum, at an angle of incidence of 25° to the normal. What is the angle between the red and violet rays that are transmitted into the vacuum? Calculate the range of angles of incidence for which red light will be transmitted across the boundary, but violet light will not be.

[8 marks]

1.3) Assume that the Earth follows a circular orbit around the Sun. Show that in a stable orbit, the kinetic energy T is related to the potential energy V by the expression $T = -\frac{1}{2}V$. Calculate the minimum energy in Joules that would be required to move the earth to an infinite distance from the sun, explaining your reasoning.

[8 marks]

1.4) Ampère's law states that the line integral of the magnetic field $\mathbf{B}$ around a closed path is proportional to the current i enclosed: $\oint \mathbf{B}.\mathbf{dl} = \mu_0 i$.

A long, straight, cylindrical wire of radius R carries a total current I. Assuming that the current is uniformly distributed inside the wire, show that

$$B = \frac{\mu_0 I r}{2\pi R^2}, \quad \text{when} \quad r < R.$$

If the wire has a radius of 2 mm and carries a current of 3 A, what is the magnitude and direction of $\mathbf{B}$ 1 mm from the centre of the wire?

[9 marks]

1.5) An electric dipole consists of two equal and opposite charges of magnitude q, separated by a small distance $2l$. Define the dipole moment μ.

Write down an expression for the electrostatic potential at a point $(r,\ \theta)$, where r is the distance from the mid-point of the dipole and θ is the angle measured relative to the axis of the dipole.

Show that when $r >> 2l$,

$$V(r,\ \theta) \approx \frac{\mu \cos\theta}{4\pi\epsilon_0 r^2}$$

where μ is the dipole moment.

[8 marks]

SECTION B – Answer TWO questions

2a) Describe the conditions under which the interference of two beams of light may be observed.

[3 marks]

b) Two beams of light are represented by sine waves of equal amplitude A and equal wavenumber k. They are combined with a path difference of l.

i) Show that the time-independent amplitude of the sum of the waves is $2A\cos(kl/2)$.

ii) What are the values of l for the maximum and for the minimum intensities of the combined waves?

iii) When only *one* wave is used, the observed intensity is I_0. Show that the maximum intensities of the combined beams is $4I_0$. How is this consistent with the conservation of energy?

[10 marks]

c) A plane wave of monochromatic light of wavelength λ falls on a pair of narrow parallel slits, whose centres are separated by a distance d. The interference pattern is observed at a large distance $L >> d$ from the slits. Show that the maxima in the intensities are observed at angles θ_m, measured from the straight-through direction, where

$$d\sin\theta_m = m\lambda, \quad m = 0, \pm 1, \pm 2 \ldots$$

[5 marks]

d) The light has a wavelength of 541 nm, and the separation of the slits is $10\,\mu$m. Calculate the maximum value of m.

[5 marks]

e) A block of material with parallel sides is placed near one of the slits so that the light from that slit travels through a thickness $t = 1\,\mu$m of the material (refractive index $n = 1.3$). What is the change in the optical path length of the light from the slit as a result of inserting the material? At what angle will the first interference maximum now be observed?

[8 marks]

3) The Gauss law of electrostatics for the electric field $\mathbf{E}$ may be expressed as

$$\int_S \mathbf{E}.\mathbf{dS} = \sum_i \frac{q_i}{\epsilon_r \epsilon_0},$$

where the sum is over all the charges q_i in the closed surface S, ϵ_0 is the permittivity of free space, and ϵ_r is the relative permittivity of the material in the surface.

a) Use the Gauss law to show that the electric field between the plates of a parallel plate capacitor is given by

$$E = \frac{\sigma}{\epsilon_r \epsilon_0}$$

where A is the area of one of the plates, and σ is the charge per unit area on the plate.

Hence show that the capacitance C of the capacitor is

$$C = \frac{\epsilon_r \epsilon_0 A}{d},$$

where d is the separation pf the plates. State any assumptions that you make.

[8 marks]

b) When the potential difference between the plates is V, show that the energy W stored in the capacitor is

$$W = \frac{1}{2} C V^2.$$

[4 marks]

c) A parallel plate capacitor has two plates, each of area $30\,\text{cm}^2$, separated by $0.5\,\text{mm}$.

i) Calculate its capacitance, assuming that the material between the plates is air with relative permittivity 1.0.

[2 marks]

ii) The capacitor is connected to a d.c. source of $100\,\text{V}$ until it is fully charged, when the capacitor is then disconnected. Calculate the charge on each plate, and also the energy stored in the capacitor.

[4 marks]

iii) When a sheet of insulating material with a relative permittivity of 3.0 is inserted between the plates, the charge on each plate remains the same but the potential difference changes.

Calculate the new potential difference.

[4 marks]

What is the energy now stored in the capacitor?

[4 marks]

Explain how your result is consistent with the conservation of energy.

[4 marks]

4) a) The e.m.f. v_e generated in a coil with a self-inductance L when there is a rate of change of current $\mathrm{d}i/\mathrm{d}t$ is

$$v_e = -L\mathrm{d}i/\mathrm{d}t.$$

What is the physical significance of the negative sign?

[3 marks]

b) At time $t = 0$, an inductor with a resistance R and self-inductance L is connected across a battery of e.m.f. V. Show that the current i at a later time t is

$$i = \frac{V}{R}\left[1 - \exp\left(-\frac{Rt}{L}\right)\right].$$

[7 marks]

c) Sketch the behaviour of i as a function of t. After a long time, the current is found to be $i = 1\,\mathrm{A}$ when $R = 10\ \Omega$. What is the e.m.f. of the battery?

[3 marks]

d) Consider a circuit in which there are two coils with inductances of $L_1 = 0.1\,\mathrm{H}$ and $L_2 = 0.05\,\mathrm{H}$ respectively. The coils have negligible resistance and no mutual inductance.

i) The coils are connected in series with a battery. Show that the two coils are equivalent to one coil of inductance $L = 0.15\,\mathrm{H}$.

[4 marks]

ii) The two coils are now connected in parallel with the battery. Show that they are now equivalent to a single coil of inductance $L = 0.033\,\mathrm{H}$.

[8 marks]

iii) The two coils are connected as in part (d ii) and the battery has an e.m.f. of 10 V. What is the total current 0.1 s after connecting the circuit?

[5 marks]