King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP1480 Fields and Waves

January 2005

Time allowed: THREE Hours

Candidates should answer all SIX parts of SECTION A, and no more than TWO questions from SECTION B. No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0 =	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0 =	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c =	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G =	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e =	1.602×10^{-19}	C
Electron rest mass	m_{e} =	9.109×10^{-31}	kg
Unified atomic mass unit	m_{u} =	1.661×10^{-27}	kg
Proton rest mass	m_{p} =	1.673×10^{-27}	kg
Neutron rest mass	m_{n} =	1.675×10^{-27}	kg
Planck constant	h =	6.626×10^{-34}	J s
Boltzmann constant	k_{B} =	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ =	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R =	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_{A} =	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP	=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0 =	1.013×10^{5}	$\mathrm{N\,m^{-2}}$

SECTION A – Answer all SIX parts of this section

1.1) Two large stars with equal masses of 9×10^{30} kg are separated by 3×10^{12} m. A smaller star of mass 10^{30} kg is located on the perpendicular bisector of the line joining the two larger stars, at a distance of 4×10^{12} m from each of the larger stars. Calculate the force on the smaller star and its acceleration.

[7 marks]

1.2) An electron orbits in a circular orbit of radius 0.053 nm around a proton. Calculate the change in potential energy when the electron is removed from its orbit and located a large distance from the proton, expressing the answer in electron volts.

[7 marks]

1.3) An electron is travelling through a vacuum with a velocity $\mathbf{v} = (2\mathbf{i} + 3\mathbf{k}) \times 10^5$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{k}$ are orthogonal unit vectors. It suddenly enters a uniform magnetic field $\mathbf{B}$ of magnitude 5 T in the $\mathbf{k}$ direction. What is the effect on the speed of the electron in the $\mathbf{k}$-direction? What is the radius of the circular motion perpendicular to $\mathbf{k}$?

[7 marks]

1.4) Ampère's law states that the line integral of the magnetic field around a closed path is proportional to the current enclosed: $\oint \mathbf{B}.\mathbf{dl} = \mu_0 I$. Use the law to find an expression for the field at a distance r from a straight, long wire that carries a current I. Calculate the magnitude of $\mathbf{B}$ at a distance of 10 cm when $I = 5$ A.

[7 marks]

1.5) Describe what is meant by *total internal reflection.* An optical fibre has a core of glass with refractive index 1.59, which is surrounded by a protective cladding of glass with refractive index 1.46. Determine the range of angles at which light in the core can hit the cladding and be internally reflected?

[7 marks]

1.6) A thin-lens magnifying glass is placed 25 mm above an object. A virtual image is observed at 5 times magnification. What is the focal length of the lens?

[7 marks]

SECTION B – Answer TWO questions

2a) The Gauss law of electrostatics may be expressed as

$$\int_S \mathbf{E}.d\mathbf{s} = \sum_i \frac{q_i}{\epsilon_r \epsilon_0}.$$

Explain the meaning of this equation and define each of the quantities involved.

[8 marks]

b) A coaxial cable consists of a copper wire, of radius r_1, with an outer conductor of radius r_2 separated from it by a material of permittivity $\epsilon_r \epsilon_0$. Use the Gauss law to show that the capacitance of the cable per unit length is

$$C = \frac{2\pi \epsilon_r \epsilon_0}{\ln(r_2/r_1)}.$$

[8 marks]

c) The coaxial cable has $r_1 = 0.5$ mm, $r_2 = 3.0$ mm, and $\epsilon_r = 2.1$. A 1 m length of the cable has its central wire connected to a d.c. source of +100 V, and its outer conductor connected to earth.

i) Calculate the capacitance of the metre length of the cable.

[4 marks]

ii) Calculate the magnitude of the charge on the central wire. How many electrons have been removed from it?

[5 marks]

iii) Calculate the electrostatic energy that is stored in the cable.

[5 marks]

3a) State Newton's law of gravitation for the force between two point masses.

[3 marks]

Using the equation for the force, show that the gravitational potential U at any point a distance r from a mass M is $U = -GM/r$.

[5 marks]

b) The gravitational potential at a point *outside* a sphere can be calculated by assuming that its mass is concentrated at its centre. The potential anywhere *inside* a thin spherical shell is constant and equal to its value at the surface of the shell.

A sphere of radius R has a uniform mass-density ρ. Show that the gravitational potential U at a distance s from the centre ($s < R$) is

$$U = 2\pi G\rho \left[\frac{s^2}{3} - R^2\right].$$

[12 marks]

c) Derive an expression for the force F on a small mass m when the mass is at a distance s ($s < R$) from the centre of the sphere.

[5 marks]

d) A spherical, homogeneous planet has a straight tunnel drilled through it along a diameter. A particle is released from the surface of the planet into the tunnel. Describe the subsequent motion of the particle, assuming that only the gravitational force is acting.

[5 marks]

4a) Describe the conditions under which the interference of two beams of light may be observed.

[6 marks]

b) A plane wave of monochromatic light of wavelength λ falls on a pair of parallel slits, whose centres are separated by a distance d. Show that bright fringes are observed at angles θ_n, measured from the straight-through direction, where

$$d \sin\theta_n = n\lambda, \quad n = 0, 1, 2\ldots$$

[8 marks]

c) The interference pattern for light of wavelength 541 nm is observed on a screen located 1 m from the slits. The first bright fringe is a distance of 1 mm from the central fringe. Calculate the separation of the slits.

[6 marks]

d) When only *one* slit is used, the intensity observed in the central part of the screen is approximately constant at a value of I_0. Show that the maximum intensity with *two* slits is $4I_0$. How is this consistent with the principle of conservation of energy?

[6 marks]

Hint: $\sin A + \sin B = 2\cos\left[\frac{1}{2}(A-B)\right]\sin\left[\frac{1}{2}(A+B)\right].$

e) Describe qualitatively how the intensity of the interference pattern in a double-slit experiment is affected by diffraction through each of the slits.

[4 marks]