# King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

**B.Sc. EXAMINATION**

**CP/1480 Fields and Waves**

**Summer 2003**

**Time allowed: THREE Hours**

**Candidates should answer SIX parts of SECTION A, and TWO questions from SECTION B.**

**The approximate mark for each part of a question is indicated in square brackets.**

**You must not use your own calculator for this paper.**
**Where necessary, a College calculator will have been supplied.**

**TURN OVER WHEN INSTRUCTED**

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| | |
|---|---|
| Permittivity of free space | $\epsilon_0 = 8.854 \times 10^{-12}$ F m$^{-1}$ |
| Permeability of free space | $\mu_0 = 4\pi \times 10^{-7}$ N A$^{-2}$ |
| Gravitational constant | $G = 6.670 \times 10^{-11}$ N m$^2$ kg$^{-1}$ |
| Mass of electron | $m_e = 9.109 \times 10^{-31}$ kg |
| Charge of electron | $e = -1.602 \times 10^{-19}$ C |
| Mass of proton | $m_p = 1.673 \times 10^{-27}$ kg |
| Radius of the Earth | $R = 6380$ km |
| Mass of the Earth | $M_E = 5.987 \times 10^{24}$ kg |

## SECTION A – Answer SIX parts of this section

1.1) In a hydrogen atom the electron and proton are attracted to each other by gravitational and electrostatic forces. Calculate the ratio of these forces.

[7 marks]

1.2) An electric dipole consists of two point charges of $+e$ and $-e$ separated by a distance of 0.2 nm. The dipole is placed in a uniform electric field of strength $E$. Write down an expression for the force on each charge resulting from the electric field. Calculate the torque when the dipole is at an angle of 30° to a field $E = 10^5$ V m$^{-1}$, and the charges have the magnitude of the electronic charge.

[7 marks]

1.3) The mass of the Moon is about 1/81 that of the Earth, and its radius is about 1/4 of the Earth's. Using the data at the head of the paper, calculate the acceleration due to gravity at the surface of the Moon.

[7 marks]

1.4) An electron travels with a velocity $\mathbf{v} = 2 \times 10^6\mathbf{i} - 2 \times 10^6\mathbf{j}$ m s$^{-1}$ in a magnetic field $\mathbf{B} = 0.15\mathbf{i} - 0.02\mathbf{j}$ Tesla. What is the (vector) force on the electron?

[7 marks]

1.5) A capacitor of capacitance $C$ has charges of $\pm Q_0$ on its plates. It is connected at time $t = 0$ across a resistor of resistance $R$. Show that the charge at time $t$ is $Q = Q_0 \exp(-t/RC)$.

[7 marks]

1.6) A magnetic dipole can be imagined to consist of two unlike monopoles of equal magnitude, separated by a small distance. Write down the potential produced by the two poles at a point $(r,\ \theta)$, where $r$ is the distance from the centre of the dipole and $\theta$ is the direction of the point relative to the axis of the dipole. Show that when $r$ is large,

$$V(r,\ \theta) \approx \frac{\mu \cos\theta}{4\pi\mu_0 r^2}$$

where $\mu$ is the magnetic dipole moment.

[7 marks]

1.7) Describe what is meant by *total internal reflection.* A cube of glass of refractive index 1.6 is surrounded by a vacuum. A beam of light travels inside the glass, and hits the surface at an angle $\theta$ to the perpendicular. Calculate the minimum angle at which the light will be totally internally reflected.

[7 marks]

1.8) An object is located at a distance $u$ from a thin lens of focal length $f$, and its image is formed at a distance $v$ on the other side of the lens. Show that $1/u + 1/v = 1/f$. An object of height 20 mm is placed 500 mm from a lens of focal length 200 mm. What is the height of the image?

[7 marks]

## SECTION B – Answer TWO questions

2)

a) State Newton's law of gravitation for the force between two point masses.

[3 marks]

Using the equation for the force, show that the gravitational potential $U$ at any point a distance $r$ from a mass $m$ is $U = -Gm/r$.

[6 marks]

b) The gravitational potential at a point *outside* a sphere can be calculated by assuming that its mass is concentrated at its centre. The potential *inside* a spherical shell is equal to its value at the surface of the shell.

A sphere of radius $R$ has a uniform mass-density $\rho$. Show that the gravitational potential $U$ at a distance $s$ from the centre ($s < R$) is

$$U = 2\pi G\rho \left[\frac{s^2}{3} - R^2\right].$$

[12 marks]

c) Show that the force $F$ on a mass $M$ at a distance $s$ ($s < R$) from the centre of the sphere is given by

$$F = -\frac{4}{3}\pi G\rho M s.$$

[3 marks]

d) What is the fractional change in the weight of a gold miner working at a depth of 5 km below the surface of the Earth compared to his weight at the surface?

[You may assume that the Earth has a uniform density.]

[6 marks]

3)

a) The e.m.f. $v_e$ generated in a coil with a self-inductance $L$ arising from a rate of change of current $\mathrm{d}i/\mathrm{d}t$ is $v_e = -Ldi/dt$. What is the physical meaning of the negative sign?

[3 marks]

b) An inductor with a resistance $R$ and self-inductance $L$ is connected across a battery of e.m.f. $V$ at time $t = 0$. Show that the current $i$ at a time $t$ is

$$i = \frac{V}{R}\left[1 - \exp\left(-\frac{Rt}{L}\right)\right].$$

[10 marks]

c) Show that the work $W$ done by the battery during the time $t$ after connection is

$$W = \frac{V^2}{R}\left[t + \frac{L}{R}\exp\left(-\frac{Rt}{L}\right) - \frac{L}{R}\right].$$

[10 marks]

d) After a long time, the current is found to be $i = 1$ A when $R = 1\ \Omega$ and $L = 0.1$ H. What is the work done between times $t = 0$ and $t = 10$ s?

[7 marks]



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4)

a) What is meant by the *interference* of two beams of light?

[5 marks]

b) Explain the meaning of the term *temporal coherence*.

[5 marks]

A beam of monochromatic light of wavelength $\lambda$ passes through two narrow parallel slits that are separated by a distance $d$. The intensity $I$ of the light is measured at an angle $\theta$ from the straight-through direction.

c) Show that

$$I \propto \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right).$$

[You may assume that $\cos(a+b)+\cos(a-b) = 2\cos a \cos b$.]

[12 marks]

d) The interference pattern produced by light of wavelength 500 nm is observed on a screen placed 2 m from the slits. Successive maxima in the interference pattern are separated by 5 mm. What is the separation of the slits?

[8 marks]

5) The magnetic field at a vector distance $\mathbf{r}$ from a short piece of wire of vector length $\mathrm{d}\mathbf{l}$ carrying a current $i$ is

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i\mathrm{d}\mathbf{l}\times\mathbf{r}}{r^3}.$$

a) Show that the component of the magnetic field along the axis of a circular coil of $n$ turns of wire at a distance $x$ from the centre of the coil is

$$\mathbf{B} = \frac{\mu_0}{2}\frac{nia^2}{(x^2+a^2)^{3/2}},$$

where $a$ is the radius of the coil.

[8 marks]

b) Two equally-sized, coaxial circular coils carry the same current in the same direction around the coils. They are separated by a distance $s$. Show that the magnetic field between the coils at a distance $x$ from one of the coils is

$$B = \frac{\mu_0 ina^2}{2}\left[\frac{1}{(a^2+x^2)^{3/2}} + \frac{1}{[a^2+(s-x)^2]^{3/2}}\right].$$

[6 marks]

Sketch the magnitude of $B$ as a function of $x$, and explain why this configuration of two coaxial coils is useful.

[4 marks]

A small magnetic dipole of magnetic moment $\mathbf{m}$ is aligned along the axis of the coils and placed at the point $x$. Derive an expression for the force on the dipole when $x = s/2$.

[12 marks]