King's College London

UNIVERSITY OF LONDON

M.Sci. EXAMINATION

CP/4474 Physics at the Nanoscale

Summer 2005

Time allowed: THREE Hours

Candidates must answer THREE questions.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	$=$	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$=$	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	$=$	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	$=$	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	$=$	1.602×10^{-19}	C
Electron rest mass	m_e	$=$	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	$=$	1.661×10^{-27}	kg
Proton rest mass	m_p	$=$	1.673×10^{-27}	kg
Neutron rest mass	m_n	$=$	1.675×10^{-27}	kg
Planck constant	h	$=$	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_B	$=$	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	$=$	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	$=$	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	$=$	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		$=$	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	$=$	1.013×10^{5}	$\mathrm{N\,m^{-2}}$
Bohr magneton	μ_B	$=$	9.274×10^{-24}	$\mathrm{J\,T^{-1}}$

GaAs has a direct energy gap of 1.43 eV, a relative permittivity of $\epsilon = 10.9$, and a lattice constant of 0.565 nm. Its electron, heavy hole and light hole effective masses are 0.072, 0.6 and 0.15 times the free electron mass m_0.

Answer THREE questions

1)

a) A thin *strained* layer of one semiconductor is grown inside a second semiconductor of larger energy gap and smaller lattice spacing. Describe, and explain qualitatively, the effect of the strain on the energy gap of the thin layer.

[3 marks]

b) Explain why there is a practical limit to the thickness of a strained layer of semiconductor.

[3 marks]

c) A quantum well of GaAs is sandwiched between thick layers of an AlGaAs alloy which is *lattice-matched* to the GaAs. Electrons and holes are assumed to have free-particle behaviour for motion in the x, y plane of the layer, and wavefunctions of the form $\sin(\pi z n/a)$ where n is an integer. Show that light travelling in the direction perpendicular to the plane of the layer can only be absorbed in the layer by creating electrons and holes with the same values of n.

[6 marks]

Data for GaAs are given at the head of the paper.

d) Assuming that a particle of mass m in an infinitely deep potential well of width a has energy levels

$$E = \frac{h^2}{8ma^2} n^2, \quad n = 1, 2 \ldots,$$

estimate, giving your answer in eV, the lowest photon energy ($h\nu$) of the light that can be absorbed in a GaAs layer of thickness 5 nm.

[3 marks]

e) What is the rate of change of the photon energy $h\nu$ with the thickness a of the layer? A layer is grown on a $< 001 >$ crystallographic surface of the substrate. Its thickness in places is 5 nm, but in other places it is greater by an amount equal to the smallest separation of two Ga atoms in the $< 001 >$ direction. What is the fractional variation in $h\nu$ across the plane?

[3 marks]

2)

a) A lattice-matched quantum layer of $\mathrm{Ga}_x\mathrm{In}_{1-x}\mathrm{Sb}$ is buried inside AlSb. The AlSb is doped so that the layer is n-type. A magnetic field is applied in the z direction, defined as being perpendicular to the plane of the layer. The separation of the Landau levels (from a spin-up state to a spin-up state) is 40.6 meV at a field of 21 T. Calculate the value of the effective mass of the electrons at that field.

[5 marks]

b) Show that, per unit area of the layer, there are $g(E)\,\mathrm{d}E = m^*/(\pi\hbar^2)\,\mathrm{d}E$ electron states in the energy range E to $E + \mathrm{d}E$, where m^* is the effective mass of the electrons. If the layer contains $n_A = 5 \times 10^{10}$ cm^{-2} electrons, calculate the magnetic field B_1 at which the lowest pair of Landau levels (i.e. with spin up and spin down) is filled.

[5 marks]

c) With the magnetic field on the z axis, an electrical current flows through the layer in the x direction and a Hall voltage is generated along the y axis. The transverse resistance of the layer is defined as $\rho_T = E_y/J_x$ where E_y is the electric field created by the Hall effect and J_x is the current flowing divided by the width of the layer in the y direction. Show that at the field B_1, ρ_T does not depend on the doping level n_A, on the effective mass m^* or on the dimensions of the layer. Calculate the value of ρ_T for the quantum layer at the field B_1. What is the practical value of this result?

[10 marks]

3)

a) Show that in a quantum wire there are $g(E)\,\mathrm{d}E$ electron states per unit length bewteen energies E and $E+\ \mathrm{d}E$ where

$$g(E) = \frac{1}{h}\sqrt{\frac{2m^*}{E}},$$

where m^* is the effective mass of the electrons.

[4 marks]

b) Defining the group velocity as $v_g = \mathrm{d}\omega/\mathrm{d}k$, show that the product of the v_g and $g(E)$ is independent of the energy of the electrons and the properties of the quantum wire.

[4 marks]

c) Justify, qualitatively, the definition of the current operator

$$\hat{J} = \frac{e\hbar}{m^*}\frac{\partial}{\partial x}.$$

Show that, for a state $\psi(x) = \exp(ikz)$ representing an electron moving freely along the length of a quantum wire, the eigenvalue of $\hat{J}$ has a magnitude ev_g.

[5 marks]

d) Show that when the quantum wire connects a reservoir of charge to a sink of charge with a potential difference $|V|$ between them, the electrical conductance is quantised in units proportional to S where $S^{-1} = 25.2\ \mathrm{k}\Omega$.

[5 marks]

e) Describe the conditions under which one might observe these quantised steps.

[2 marks]

4)

a) Explain the relaxation oscillations in the building up of charge (and voltage) across an ultra small tunnel junction. [4 marks]

b) Estimate the temperature at which a 20×20 nm^2 parallel-plate tunnelling capacitor with a thickness of 2 nm of insulating Al_2O_3 layer is expected to show Coulomb blockade. What would be the threshold voltage? ($\epsilon_r = 2$ for Al_2O_3). [6 marks]

c) A certain conductor has the shape of a strip with the dimensions $L_x = 1000$ nm in the direction of the electrical current, $L_y = 10$ nm and $L_z = 2$ nm in the transverse directions. The elastic scattering rate for the conduction electrons, $\tau^{-1} = 2 \times 10^{14}$ s^{-1}; the phase breaking rate, τ_ϕ^{-1}, changes with temperature according to the law $\tau_\phi^{-1} = AT^3$ s^{-1}; $A = 3 \times 10^8$ s^{-1} K^{-3}; and the Fermi velocity, $v_F = 2 \times 10^6$ m s^{-1}.

i) Calculate the temperature at which the conductor reaches the 1D regime with respect to the weak localisation effect. [6 marks]

ii) Sketch the dependence of the resistance of the conductor in the 1D weak localisation regime on magnetic field for two orientations: (1) parallel to the $0z$ axis, and (2) parallel to the $0y$ axis. [4 marks]

5)

a) Describe the Dolan bridge technique for nanofabrication of sub-micron tunnel junctions. [4 marks]

b) Explain the mechanism for the proximity effect which occurs during electron beam lithography. Describe how corrections may be applied to minimise the results of the proximity effect. How may the sample configuration be altered to minimise the effect? [6 marks]

c) Describe the negative resist process for patterning a thin film using electron-beam lithography. Describe briefly the phenomenon taking place during the exposure of negative resist with an electron beam. [5 marks]

d) Sketch the DC diode system for sputter deposition. How can the system be converted to generate plasma dry etch for use during the negative-resist lithography process? [5 marks]