King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3640 Modern Topics in Astrophysics

Summer 1998

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Speed of light, $c = 3.0\times10^{8}\ \mathrm{m\,s^{-1}}$

Gravitational constant, $G = 6.7\times10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$

1 parsec, $1\ \mathrm{pc} = 3.1\times10^{16}\ \mathrm{m}$

1 year, $1\mathrm{y} = 3.2\times10^{7}\ \mathrm{s}$

1 metric tonne, 1 tonne = 1000 kg

Neutrino cross sections:

$\nu_e + e^- \to \nu_e + e^-$ $\quad 9.3\times10^{-49} E\ \mathrm{m}^2$

$\nu_\mu + e^- \to \nu_\mu + e^-$ $\quad 1.5\times10^{-49} E\ \mathrm{m}^2$

$\nu_\tau + e^- \to \nu_\tau + e^-$ $\quad 1.5\times10^{-49} E\ \mathrm{m}^2$

In the above expressions for the cross-sections, E is the neutrino energy in MeV.

SECTION A. Answer any **SIX** parts of this section.

1.1 State the cosmological principle, and state what the principle implies about the structure of the universe. Use the principle to deduce the Hubble Law in terms of the Hubble parameter H and the length scale R.

[7 marks]

1.2 The Friedmann equation is

$$\left(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 + \frac{kc^2}{R^2} = \frac{8\pi G}{3}\rho.$$

Explain the meaning of the symbols which occur in this equation. Define Ω, the ratio of matter density to the closure density, in terms of the Hubble parameter and estimate the present value of the closure density given that $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$.

[7 marks]

1.3 What are the properties of the cosmic microwave background radiation which lead to the horizon problem in cosmology?

[7 marks]

1.4 It is believed that the ratio of the number of baryons in the universe to the number of photons is about 10^{-9}. Explain briefly the reasons for this belief and how this imbalance may have arisen in the evolution of the universe.

[7 marks]

1.5 Draw the lowest-order Feynman diagrams for the processes

$$\nu_e + e^- \to \nu_e + e^-$$

$$\nu_\mu + e^- \to \nu_\mu + e^-.$$

Hence explain, qualitatively, why the two processes have different cross sections.

[7 marks]

1.6 A star at a distance of 20 kpc from the Earth is observed to undergo a supernova explosion, and a corresponding burst of neutrinos, with mean energy 12 MeV, is detected. *Estimate*, stating any assumptions, a lower limit on the neutrino rest lifetime.

[7 marks]

1.7 The following are headlines from the science pages of national daily newspapers:

Plasma opacity experiments at Rutherford Appleton Laboratory show that Standard Solar Model has "serious inadequacies"

"Sun powered by central black hole" claims British boffin

What would be the significance for the solar neutrino problem if either of these headlines were true? State, giving one reason in each case, whether or not the headlines are plausible.

[7 marks]

1.8 If neutrinos were found to undergo vacuum oscillations on a length scale small compared to interstellar distances, how would this affect the signals observed in supernova neutrino detectors?

[7 marks]

SECTION B. Answer any **TWO** questions from this section.

2 a) Modify the Friedmann equation given in question 1.2 to include the cosmological constant $\Lambda/3$. Hence deduce the form of the equation appropriate to model a flat, matter dominated universe with a non-zero cosmological constant.

[6 marks]

b) Integrate the equation to determine the time dependence of the scale parameter R, namely,

$$R = \left(\frac{3\mu}{\Lambda}\right)^{1/3} \sinh^{2/3}\left(\frac{3}{2}\sqrt{\frac{\Lambda}{3}}\, t\right),$$

where μ is a constant. [You may need to make the substitution $R^3 \propto \sinh^2\theta$ where θ is a dummy variable.]

[10 marks]

c) Hence show that the dimensionless quantity Ht, where H is the Hubble parameter, is given by

$$Ht = x \coth\frac{3}{2}x, \quad \text{where } x = \sqrt{\frac{\Lambda}{3}}\, t.$$

[7 marks]

d) Given that the solution of this equation when $H_0 = 70\,\text{km s}^{-1}\,\text{Mpc}^{-1}$ and $t_0 = 15\times10^9$ y is $x = 0.96$, show that the addition of the cosmological constant to the Friedmann equation is equivalent to an additional density of $7.4\times10^{-27}\,\text{kg m}^{-3}$. How does this compare with the present value of the closure density?

[7 marks]

3 Write an essay on the thermal history of the universe in the context of the standard model, giving details of the important stages in the evolution of the universe.

[30 marks]

SEE NEXT PAGE

4. a) Briefly describe the evolution of the star Sk-69 202 up to the time it exploded as supernova SN1987A, indicating any differences from previous supernova observations.

[8 marks]

b) Discuss the role played by neutrinos and antineutrinos before and during the supernova explosion.

[8 marks]

c) Explain why the detectors in operation at the time of SN1987A were more sensitive to antineutrinos than to neutrinos.

[6 marks]

d) Twelve events, with a mean energy of 16.5 MeV, ascribed to antineutrino interactions from SN1987A, were observed in the 2140 tonne Kamiokande detector. *Estimate*, stating any assumptions, the detector mass required to detect 2000 antineutrino interactions from a supernova at a distance one fiftieth that of SN1987A and emitting antineutrinos at 50% higher mean energy.

[6 marks]

e) Imagine that funds are provided to build such a detector for use in 3rd year undergraduate projects in the teaching laboratory on the second floor of the main building at King's College. Discuss whether the money will be well-spent on this plan.

[2 marks]

5 The existence of the *solar neutrino problem* and the *solar neutrino paradox* is inferred from data collected by four detectors.

a) What is meant by the (i) solar neutrino problem and (ii) the solar neutrino paradox?

[5 marks]

b) Discuss the limitations of the four detectors used to date in the study of solar neutrinos.

[6 marks]

c) Several detectors which, it is hoped, will overcome these limitations, are under construction or are planned. One such is the Sudbury Neutrino Observatory (SNO). Briefly describe this detector, paying particular attention to the improvements over current detectors.

[9 marks]

d) SNO will start recording data soon. Suppose that, after the first year's operation, the data rates (events per year) in the following table are obtained. The expected rates for the Standard Solar Model (SSM) are also given. The observed rates do not vary significantly with time.

Process	SSM rate	Observed rate
$\nu_e + d \rightarrow p + p + e^-$	10^4	$\sim 3\times10^3$
$\nu + e^- \rightarrow \nu + e^-$	10^3	~ 450
$\nu + d \rightarrow \nu + n + p$	5×10^3	$\sim 5\times10^3$

Give, stating your reasons, a plausible and, as far as possible, quantitative, explanation of the observed rates compared to those expected.

[10 marks]