King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3480 Electromagnetism and Optics 2

Summer 1998

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Permittivity of free space	ϵ_0	$= 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$= 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Speed of light in free space	c	$= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$

SECTION A – Answer SIX parts of this section

1.1) State the complex form of Maxwell's equations for the electromagnetic field, defining all of the symbols used.

[7 marks]

1.2) In electromagnetic theory it is convenient to introduce a complex refractive index $n = \chi - i\kappa$. Explain, qualitatively, the physical significance of the parameters χ and κ.
Give a formula for the wavelength of an electromagnetic wave of frequency f Hz propagating in a medium having this complex refractive index.

[7 marks]

1.3) Show that a distribution of free-charge density ρ, within a linear, homogeneous medium of permittivity ε and conductivity σ, decays exponentially with a time constant $\tau = \varepsilon/\sigma$.

[7 marks]

1.4) Define the terms *dielectric approximation* and *metallic approximation* in relation to an electromagnetic wave of angular frequency ω propagating in a material having a permittivity ε and a conductivity σ.
A wave of frequency 10 kHz propagates in sea water for which the relative permittivity $\varepsilon_r = 81$ and the conductivity $\sigma = 4.0\,\mathrm{S\,m^{-1}}$. Prove that the metallic approximation applies in this situation.

[7 marks]

1.5) State the two principal ideas that underlie the Abbe theory of image formation. Use these ideas to explain when it would be possible for the image to be a faithful copy of the object in a single-lens optical system.

[7 marks]

1.6) Explain how the holographic recording of a scene differs from a conventional photographic image of the same scene.

[7 marks]

1.7) Describe, with the aid of a diagram, how a *pseudoscopic* image can be formed when reconstructing a holographic image.
Explain briefly what is meant by the term *pseudoscopic*.

[7 marks]

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1.8) State whether the following statements are true or false, and give your reasoning.

a) The resolution of a holographic image recording will be improved by using a film emulsion with a smaller grain size.

b) Illumination by a laser source is an essential requirement for recording a hologram with visible light.

c) Successful laser operation relies on the fact that photons produced by stimulated emission are in phase with the stimulating photons.

[7 marks]

SECTION B – Answer TWO questions

2) Explain the meaning of the terms *plane electromagnetic wave* and *state of polarisation.*

[6 marks]

Define the *Poynting vector.* What precaution must you take in evaluating the time-averaged Poynting vector for an electromagnetic field which is expressed in the complex (or cissoidal) form?

[4 marks]

The electric vector of an electromagnetic field in free space is given as

$$E_x = 0; \quad E_y = E_0 \exp\left[i(\omega t - \beta x)\right]; \quad E_z = E_0 \exp\left[i(\omega t - \beta x - \pi/4)\right]$$

where E_0, ω and β are constants.

a) What is the state of polarisation of this wave?

[4 marks]

b) Use Maxwell's equation for curl $\mathbf{E}$ to calculate the magnetic field strength $\mathbf{H}$ of this wave and draw a sketch to illustrate the polarisation of the magnetic field.

[10 marks]

c) Derive a formula for the time-averaged Poynting vector of the wave.

[6 marks]

3) With reference to sketches, explain what is meant by a *Hertzian dipole* antenna and a *Fitzgerald* (or magnetic) *dipole* antenna.

[4 marks]

Define the *charge* and *current* moments of the Hertzian dipole and derive the relation between them.

[3 marks]

It is given that the magnetic vector potential $\mathbf{A}$ for a cissoidal current density of amplitude $\mathbf{j}_0$ distributed through a volume v is

$$\mathbf{A}(\mathbf{r}, t) = \mu_0 \int \frac{\mathbf{j}_0 \exp\left[i\omega(t - r/c)\right]}{4\pi r} dv.$$

Consider a z-directed Hertzian dipole of current moment $I\delta z$ located in free space at the point $(0, 0, 0)$. Show that the magnetic vector potential is also z-directed and given by

$$A_z(\mathbf{r}, t) = \mu_0 I\delta z \frac{\exp\left[i(\omega t - kr)\right]}{4\pi r}$$

where $k^2 = \omega^2 \mu_o \varepsilon_0$.

[3 marks]

State formulas for the spherical polar (r, θ, ϕ) components of $\mathbf{A}$ and hence prove that the magnetic field strength produced by this source only has a ϕ-component which is given by

$$H_\phi = \frac{I\delta z}{4\pi} \left[\frac{ik}{r} + \frac{1}{r^2}\right] \sin(\theta) \exp\left[i(\omega t - kr)\right].$$

For this problem you may assume that

$$\nabla \times \mathbf{A} = \hat{\phi}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r)\right]$$

where $\hat{\phi}$ is a unit vector in the ϕ direction.

[12 marks]

Identify the radiation term in the magnetic field formula and state the physical significance of this term.

[4 marks]

Which spherical-polar component of the electric field $\mathbf{E}$ is dominant in the radiation zone and what is the direction of the Poynting vector in this zone?

[4 marks]

4) Write down expressions that describe the rates of (a) spontaneous emission, (b) resonant absorption and (c) stimulated emission in a two-level atomic system, defining all the symbols used.

[4 marks]

Explain what is meant by a *population inversion*, and show that a population inversion is a necessary requirement for successful laser operation. Explain why it is easier to sustain a population inversion in a 4-level laser than in a 3-level laser.

[8 marks]

Describe three benefits that the use of a resonant cavity brings to the operation of a laser. List three effects that can lead to a loss of light intensity within the resonant cavity of a working laser.

[9 marks]

Show that for a resonant cavity of length L the frequency difference between consecutive longitudinal resonant modes is $\Delta\nu = c/2nL$, where c is the free-space velocity of light, and n is the refractive index of the medium within the cavity.

[3 marks]

A laser with a resonant cavity that is 400 mm long emits light at a wavelength of 560 nm with a Doppler-broadened transition width of 1.2 GHz. If the refractive index of the lasing medium is 1.76, estimate the number of resonant longitudinal modes that will be sustained by the laser.

[4 marks]

Suggest one change to the design of the laser that would result in only a single resonant longitudinal mode being sustained by the laser.

[2 marks]

5) Describe what is meant by *spatial filtering* in the context of image formation in an optical system.

[4 marks]

Consider a weak phase object with a complex transmission function of the form $f(x) = \exp[i\phi_0 \cos(2\pi x/d)]$, where ϕ_0 is a constant, and $\phi_0 \ll 1$. With the aid of a diagram, describe the Zernike method of phase contrast imaging, and derive an expression for the intensity distribution in the resulting image, stating clearly any assumptions that you have made. Show that the intensity contrast in the image plane is $2\phi_0$

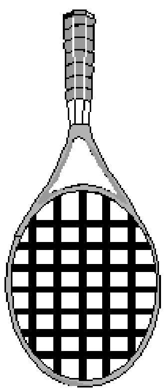
[13 marks]

Given that the period d of the cosine in the weak phase object above is $40\,\mu$m, calculate the minimum numerical aperture for the lens system that would still allow this period to be resolved, assuming that the wavelength of the illumination is 600 nm.

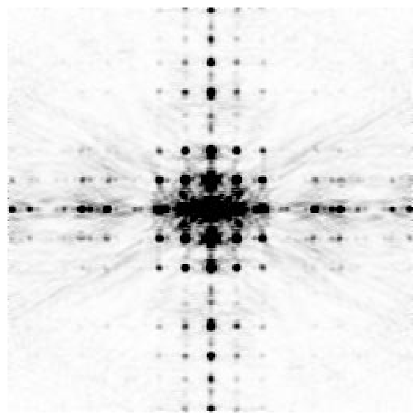
[3 marks]

Figure 5.i shows a tennis racquet. It is illuminated by a coherent monochromatic plane wave, and the intensity of the diffraction pattern in the Fourier transform plane of a two-lens imaging system is shown in Figure 5.ii. A number of different masks are used to obstruct some parts of the diffraction pattern, and the effect of these masks on the pattern is shown in Figure 5.iii. The resulting images are shown in Figure 5.iv. Identify which image corresponds to each of the masked diffraction patterns, briefly stating the reasons for each of your choices.

[10 marks]

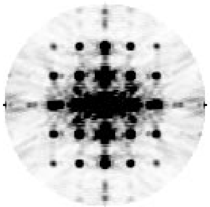


5.i Object

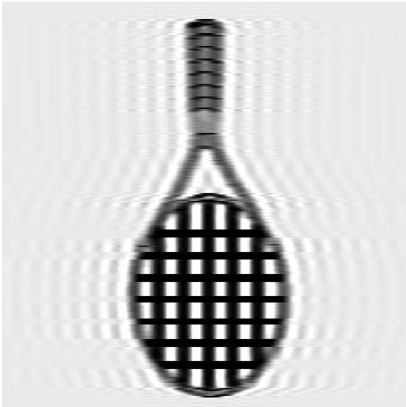


5.ii Diffraction pattern

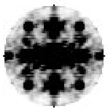
*Note: The contrast in Figures 5.*ii *and 5.*iii *has been reversed: higher values correspond to darker tones, low values appear white. In the other figures in this question, white denotes a high value, and black a low value.*



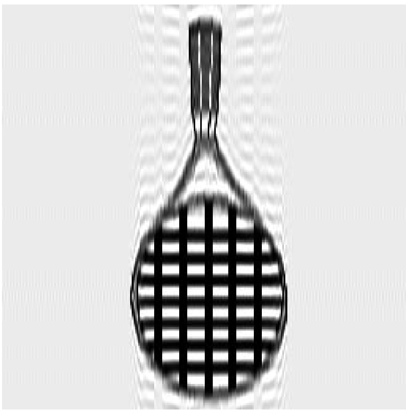
5.iii Masked pattern 1



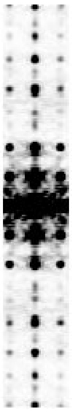
5.iv Image A



5.iii Masked pattern 2

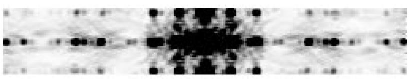


5.iv Image B

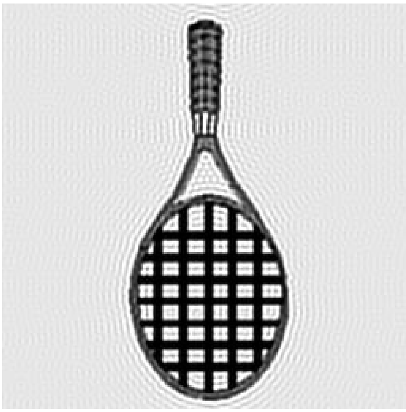


5.iii Masked pattern 3

5.iv Image C



5.iii Masked pattern 4



5.iv Image D

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