King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/3480 Electromagnetism and Optics 2

Summer 1997

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Permittivity of free space	ϵ_0	$= 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$= 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Speed of light in free space	c	$= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$

SECTION A – Answer SIX parts of this section

1.1) State the general form of Maxwell's equations for the electromagnetic field, giving the units of all the physical quantities involved.

[7 marks]

1.2) Explain what is meant by the term *Brewster angle* with reference to an appropriate sketch. How does the Brewster angle effect enable light with a linear polarisation to be produced?

[7 marks]

1.3) Define the *momentum density* of an electromagnetic field.
Derive an expression for the radiation pressure exerted by a free-space wave field incident normally on a perfectly reflecting surface.

[7 marks]

1.4) What is a standing electromagnetic wave?
Show how a standing wave may be produced by combining two propagating electromagnetic waves.

1.5) Explain briefly what is meant by a *weak phase object*.

Describe how the Schlieren method can be used to produce an image of a weak phase object.

[7 marks]

1.6) Explain briefly whether the following statements are true or false:

a) For an atomic system with 2 energy levels in thermal equilibrium, the probability of spontaneous emission of a photon will always exceed the probability of stimulated emission.

b) Stimulated emission from a medium in which there is a population inversion produces light that all has the same polarisation.

[7 marks]

1.7) Explain briefly the practical advantages of using a 4-level laser instead of a 3-level laser.

[7 marks]

1.8) The optical cavity of a semiconductor laser is 0.52 mm long, and the laser emits infra-red radiation with a mean wavelength of 1.3 μm. If the refractive index of the semiconductor material at this wavelength is 4, calculate the wavelength difference between successive resonant longitudinal modes in the light emitted by the laser.

[7 marks]

SECTION B – Answer TWO questions

2) Derive Poynting's theorem for a source-free region in a medium of permittivity ε, permeability μ and conductivity σ.

[10 marks]

Discuss the interpretation of this theorem.

[4 marks]

You may assume that, for any two vectors $\mathbf{A}$ *and* $\mathbf{B}$,

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \,.$$

The electric field in a conducting medium of permeability μ is given as

$$E_x = 0, \qquad E_y = E_0 \exp\left(-\alpha x\right) \, \cos\left(\omega t - \alpha x\right), \qquad E_z = 0$$

where E_0, ω and α are constants.
Prove that the magnetic wave-field is

$$H_x = 0, \qquad H_y = 0, \qquad H_z = \frac{\alpha E_0}{\mu\omega} \exp\left(-\alpha x\right) \, \left[\sin\left(\omega t - \alpha x\right) + \cos\left(\omega t - \alpha x\right)\right],$$

[9 marks]

and the *time-averaged* Poynting vector is x-directed and given by

$$\langle \Pi_x \rangle = \left(\frac{E_0^2 \alpha}{2\mu\omega}\right) \exp\left(-2\alpha x\right).$$

[7 marks]

3) State the general wave equation satisfied by an electromagnetic wave propagating in a medium of permittivity ε, permeability μ and conductivity σ.

[6 marks]

An electromagnetic field *in a free space region* has an electric vector given by

$$E_x = 0, \qquad E_y = E_0 \sin(\pi z/b) \exp i(\omega t - kx), \qquad E_z = 0,$$

in which E_0, b and ω are constants and x, y and z are cartesian coordinates. Use the wave equation to show that the propagation constant k is given by

$$k^2 = \left(\omega^2 - \omega_c^2\right)/c^2$$

where $\omega_c = c\pi/b$ is called the *critical* angular frequency and $c^2 = 1/(\mu_0\varepsilon_0)$.

[10 marks]

Describe the propagation characteristics of the wave for angular frequencies above and below the critical frequency ω_c. Illustrate your answer by sketching the variation of E_y for the following cases: with x, keeping t constant, and with t, keeping x constant.

[8 marks]

Show that the phase velocity ($v = w/k$) and the group velocity ($u = d\omega/dk$) satisfy the relation $vu = c^2$.

[6 marks]

4) An object is illuminated with coherent monochromatic illumination. Identify an important difference between an image of the object recorded as a conventional photograph and a holographic image of the same object.

[4 marks]

With the aid of a diagram, describe how an off-axis Fresnel hologram of a reflecting object could be recorded. Explain how the holographic image can be reconstructed, and comment briefly on the advantages and disadvantages of the off-axis geometry.

[12 marks]

A Fresnel hologram is to be recorded with coherent monochromatic light of wavelength 450 nm using a sheet of photographic emulsion that is capable of resolving 600 lines per mm and lies perpendicular to the direction of the reference wave. Calculate the maximum possible angle between the reference wave and any scattered waves that can contribute to the formation of the hologram.

[8 marks]

If the minimum resolvable features in the holographic image are $2\,\mu$m in size, calculate the numerical aperture of the lens that would be required to produce a photographic image of the same object with comparable resolution, assuming the same coherent illumination was used.

[6 marks]

5) State the two principal ideas that form the basis of Abbe's theory of image formation.

[4 marks]

A coherent monochromatic plane wave of wavelength 600 nm is travelling parallel to the optical axis of a thin converging lens of focal length 20 cm and diameter 5 cm. An object is placed on the optical axis a distance u from the lens, and the image is observed as the object distance u is changed. Explain why the image resolution becomes worse as the object distance increases.
When the object is located 50 cm before the lens, calculate the minimum diameter of a circular aperture that could be centred on the optical axis in the back focal plane of the lens without affecting the image resolution.

[7 marks]

An optical system, with the same illumination as above, consists of two lenses, each identical to the one described above, that are centred on a common optical axis, with a separation of 40 cm between them. An object is located in the front focal plane of the first lens, and it has a transmission function of the form $f(x) = [0.5 + 0.25 \sin(2\pi x/d)]$, where $d = 0.1$ mm and x is a direction perpendicular to the optical axis. Show that in the back focal plane of the first lens there will be three small bright spots, and determine the separation of these spots.

[9 marks]

An absorbing mask is now placed in the back focal plane of the first lens, so that it attenuates the *intensity* of the central spot by a factor of 4, and leaves the other two spots unaffected. Derive an expression for the intensity distribution in the image plane, and show that the intensity contrast in the image plane is a factor of 1.25 higher than the intensity contrast in the object plane.

[10 marks]