Physical Constants

Permittivity of free space	ε_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^{2}\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^{3}}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$
1 eV		=	1.602×10^{-19}	J
$1\,\mathrm{MeV\,c^{-2}}$		=	1.783×10^{-30}	kg
Pion rest mass	m_π	=	139.568	$\mathrm{MeV\,c^{-2}}$

SECTION A – Answer ALL parts of this section

1.1) Neutrons are observed to decay into protons via $n \rightarrow p + e^- + \overline{\nu}_e$. Which force mediates this decay? Explain why it is impossible for the analogous decay of free protons, $p \rightarrow n + e^+ + \nu_e$, to take place. Under what circumstances can a proton decay in this way?

[5 marks]

1.2) The following interactions can all take place via the weak force. Draw all possible first-order Feynman diagrams, involving either W^- or Z^0 bosons, in each case.

(i) $\nu_e + e^- \rightarrow \nu_e + e^-$

(ii) $\overline{\nu}_e + e^- \rightarrow \overline{\nu}_e + e^-$

(iii) $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$

[6 marks]

1.3) A particle ω decays at rest via $\omega \rightarrow \pi^+ + \pi^-$, each pion having a kinetic energy of 251 MeV. Calculate the mass of the particle ω. An alternative decay mode is $\omega \rightarrow \pi^0 + \gamma$; determine the momenta, in MeV/c, of the decay products.

[6 marks]

1.4) "Grand Unified Theories" predict that protons decay, e.g., via $p \rightarrow e^+ + \pi^0$. Biological cells can, typically, withstand an absorbed dose of a few Gray, where $1\,\mathrm{Gy} = 1\,\mathrm{J\,kg^{-1}}$, but people are not killed by protons decaying in their own bodies. *Estimate*, explaining your reasoning, a lower limit for the (mean) lifetime of protons. Discuss your result.

[9 marks]

1.5) Show that the *parity* operation is equivalent to a reflection followed by a rotation through 180°. Define *helicity* and, given that neutrinos emitted in pion decay are always left handed, argue that, if the neutrino rest mass was zero, the weak interaction could not conserve parity.

[7 marks]

1.6) Briefly describe the processes by which electromagnetic and hadron calorimeters measure the energies of particles. Explain why electromagnetic calorimeters are more accurate than hadron calorimeters.

[7 marks]

SECTION B – Answer TWO questions

2) a) Explain, without reference to detailed mathematics, the reasoning which led to a formulation of relativistic quantum mechanics via the *Dirac equation*. How did the interpretation of the solutions to this equation lead to the prediction of antimatter, in particular the positron?

[10 Marks]

b) Describe the experiment in which Anderson first observed the positron. Your description should contain explanations of the roles of the super-saturated vapour, the magnetic field and the lead plate.

[6 Marks]

c) In a reconstruction of Anderson's experiment the following measurements were made of a positively charged particle X:

Initial energy of X (before the lead plate): 4.124 MeV
Final energy of X (after the lead plate): 2.114 MeV
Final momentum of X: 2.051 MeV

Determine the mass, identity and initial momentum of X.

After passing through the lead plate, X was observed to scatter (from an unseen particle S) through an angle of 69.5º, which reduced its momentum to 2.048 MeV. Determine the mass and identity of the particle S, which you may assume was initially at rest.

[14 marks]

3) a) Why was it necessary to introduce the concept of *colour* into the quark model? What is meant by the statement "only *white* or *colourless* particles are directly observable"?

[5 marks]

b) The strong force is now considered to be due to the exchange of gluons carrying colour (r, g or b) and anticolour ($\overline{r}$, $\overline{g}$ or $\overline{b}$) between quarks. Why does the strong force have a short range and lead to quark confinement, even though gluons are believed to have rest masses of zero?

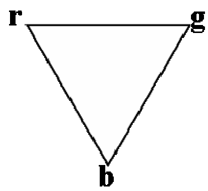
[4 marks]

c) Draw a quark Feynman diagram illustrating the first-order strong interaction between a red quark and a green quark, labelling all particles involved with their colour quantum numbers.

[5 marks]

d) By considering colour to form a fundamental triplet of SU(3), as indicated in the diagram, show that there are eight independent gluon states which form an SU(3) octet and contribute to the strong interaction. There is also one state, an SU(3) singlet, which does not contribute; explain why.

[10 marks]



e) The wave functions of three gluon states may be written as

$$\sqrt{1/2}\left(r\overline{r}+g\overline{g}\right),\ \sqrt{1/3}\left(r\overline{r}+g\overline{g}+b\overline{b}\right)\ \text{ and }\ \sqrt{1/6}\left(r\overline{r}+g\overline{g}-2b\overline{b}\right).$$

Why does $b\overline{b}$ appear differently to the other two combinations? Determine, explaining your reasoning, which of these belong to the octet and which is the singlet.

[6 marks]

4) For all parts of this question you may assume that only two neutrino types exist, electron (ν_e) and muon (ν_μ).

a) State the necessary condition, concerning mass, for neutrino oscillations to occur. Show how the matrix

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

describes the mixing between possible eigenstates of the neutrino.

[3 marks]

b) Define the system of *natural units* of speed, energy, length and time.

[4 marks]

c) A muon neutrino is generated at time $t=0$ at a particle accelerator. Show that at a later time t the probability that it is still a muon neutrino is, in natural units and in the neutrino rest frame,

$$P_\mu(t) = 1 - \tfrac{1}{2}\sin^2 2\theta\left(1 - \cos\frac{\Delta m^2}{2m}t\right)$$

and define the terms Δm^2 and m. Hence write down an expression for the probability $P_e(t)$ that, at the same time t, the neutrino has oscillated into an electron neutrino.

[9 marks]

d) Derive an expression for the time at which the probabilities $P_\mu(t)$ and $P_e(t)$ are first equal. Use this to explain the condition you stated in part a).

[6 marks]

e) In a future experiment the electron neutrino and muon neutrino rest masses are measured to be 2.6×10^{-2} eV and 3.4×10^{-2} eV, respectively. Estimate the time, in the neutrino rest frame, at which $P_\mu(t)$ and $P_e(t)$ first become equal. You may assume the current global average of experimental values for θ of 34°.

[8 marks]