King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP2621 Astrophysics

Summer 2005

Time allowed: THREE Hours

Candidates should answer all SIX parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_{e}	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_{u}	=	1.661×10^{-27}	kg
Proton rest mass	m_{p}	=	1.673×10^{-27}	kg
Neutron rest mass	m_{n}	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_{B}	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_{A}	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$
Electron volt	$1\,\mathrm{eV}$	=	1.602×10^{-19}	J
Mass of the Sun	$M_{\odot}$	=	1.989×10^{30}	kg
Radius of the Sun	$R_{\odot}$	=	6.960×10^{8}	m

SECTION A – Answer all SIX parts of this section

1.1) A star is observed to have a parallax of $p = 0.02$ arcseconds and an apparent magnitude of $m = 2.0$. What is the star's distance (in parsecs) and its absolute magnitude?

[7 marks]

1.2) The Saha equation gives the ratio of the number, N, of atoms in ionisation stage $(i+1)$ to the number in adjacent ionisation stage i. It can be written as

$$\frac{N_{i+1}}{N_i} = \frac{2k_\mathrm{B}T}{p_\mathrm{e}}\frac{Z_{i+1}}{Z_i}\left(\frac{2\pi m_\mathrm{e}k_\mathrm{B}T}{h^2}\right)^{3/2}\exp\left(-\frac{\chi_i}{k_\mathrm{B}T}\right),$$

where all the symbols have their usual meanings. Consider a stellar atmosphere composed purely of hydrogen, at a temperature of $12\,500\,\mathrm{K}$ and an electron pressure of $15\,\mathrm{N\,m^{-2}}$. Given that the partition functions for neutral and for ionised hydrogen are 2 and 1, respectively, and that the ionisation energy of hydrogen is $13.6\,\mathrm{eV}$, use the Saha equation to calculate the ratio of ionised to neutral hydrogen in this stellar atmosphere.

[7 marks]

1.3) One of the equations of stellar structure is

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3\kappa(r)\rho(r)L(r)}{64\pi\sigma r^2 T^3}.$$

Define all the terms in this equation and state what physical process the equation represents. Describe the conditions under which the equation becomes invalid.

[7 marks]

1.4) The main-sequence timescale, τ_ms, of a star may be roughly estimated as the ratio of the stellar mass, M, to its luminosity, L. In the case of the Sun, $\tau_\mathrm{ms} = 1.5 \times 10^{10}$ yrs. By deriving a relationship between τ_ms and $M[M_\odot]$, the stellar mass in solar mass units, calculate the main-sequence timescale of a $4\,M_\odot$ star.

[From stellar evolution theories, $L \propto M^{3.5}$.]

[7 marks]

1.5) Explain what is meant by *degenerate electron pressure* and by *helium flash*. State, with reasons, if a helium flash can occur in a non-degenerate gas. Stars of what mass experience a helium flash event, and at what stage of their evolution?

[7 marks]

1.6) Define the *Schwarzschild radius*, $R_{\rm sc}$, and, using semi-classical arguments, show that for a black hole of mass, M, $R_{\rm sc} = 2GM/c^2$. Hence calculate $R_{\rm sc}$ for a $3\,M_\odot$ black hole.

[7 marks]

SECTION B – Answer TWO questions

2a) i) The Boltzmann equation gives the ratio of the number, N, of atoms with energy E_b to the number with energy E_a. It can be written as

$$\frac{N(E_b)}{N(E_a)} = \frac{g_b}{g_a} \exp\left(-\frac{E_b - E_a}{k_{\mathrm{B}}T}\right),$$

where all the symbols have their usual meanings. Consider a gas of *neutral* hydrogen atoms. Given that the ionisation energy of hydrogen is 13.6 eV, use the Boltzmann equation to find the temperature at which there are 10^5 as many atoms with electrons in the ground state as there are with electrons in the first excited state.

[7 marks]

ii) Explain what is meant by Local Thermodynamic Equilibrium (LTE), naming the expressions that describe the source function, the excitation equilibrium and the ionisation equilibrium. Give an example of a region in a star where LTE is a good assumption, stating why this is the case.

[7 marks]

b) i) By considering the pressure and gravitational forces acting on a mass element, $\mathrm{d}M(r)$, of area, $\mathrm{d}A$, and thickness, $\mathrm{d}r$, at a radius, r, from the centre of a sphere of gas of density, $\rho(r)$, derive the equation of hydrostatic equilibrium

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)\rho(r)}{r^2},$$

where $M(r)$ is the mass enclosed at radius, r, and P is the pressure.

[7 marks]

ii) Consider a star to be a sphere of gas of constant density, ρ, in hydrostatic equilibrium. Use the equation of hydrostatic equilibrium and the equation of mass conservation

$$\frac{\mathrm{d}M(r)}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

to show that the central pressure, P_c, of the star is

$$P_c = \frac{3}{8\pi}\frac{GM^2}{R^4}.$$

where M and R are the mass and radius of the star, respectively. Calculate a numerical value for the Sun's central pressure assuming this model. Why is this an underestimate of the true central pressure of the Sun?

[9 marks]

3a) Sketch and label a Hertzsprung-Russell (H-R) diagram. Indicate the main sequence and the location of the Sun, and the approximate positions of the red giant, red supergiant and white dwarf stars.

[7 marks]

b) Describe the evolution of a 1 $M_{\odot}$ star from the time when hydrogen is exhausted in the core to the onset of the thermal pulses, clearly labelling all stages on the H-R diagram.

[15 marks]

c) Explain how an H-R diagram can be used to estimate the age of a star cluster.

[4 marks]

d) State the Vogt-Russell theorem for stellar evolution. For what type of star does the theorem hold?

[4 marks]

4a) i) List and briefly describe the objects that may be formed in the last stage of evolution of low-, intermediate- and high-mass stars.

[5 marks]

ii) Briefly describe the main stages of core collapse in a massive star, from the onset of photodisintegration of the heavy nuclei to the formation of a compact object.

[10 marks]

b) A white dwarf is supported against gravitational collapse by the degenerate gas pressure, P. This is proportional to a power of the gas density, ρ, according to the relationship

$$P \propto \rho^{5/3}.$$

In hydrostatic equilibrium, an approximate relationship between P and the mass, M, and radius, R, of a white dwarf is

$$P \propto \frac{M^2}{R^4}.$$

Derive a relationship between the radius of a white dwarf and its mass. Comment on the significance of this relationship.

[8 marks]

c) i) Define the *Chandrasekhar limit*, M_{ch}.

[2 marks]

ii) A simple treatment for a relativistic white dwarf gives

$$M_{\mathrm{ch}} = \frac{5.87}{\mu^2} M_{\odot}.$$

Define μ in this expression. Given that for iron, the atomic weight is 56 and the atomic number is 26, calculate M_{ch}.

[5 marks]