King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the University counting towards the award of a degree. Examinations of the University are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/2621 ASTROPHYSICS

SUMMER 2001

Time allowed: THREE HOURS

Candidates must answer any SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each section of the paper.

The approximate mark for each part of a question is indicated in the square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Constants

Electron charge (e):	1.602×10^{-19} C
Electron mass (m_e):	9.109×10^{-31} kg
Speed of light in a vacuum (c):	2.998×10^{8} m s^{-1}
Planck constant, (h):	6.626×10^{-34} J s
Boltzmann constant (k):	1.381×10^{-23} J K^{-1}
Gravitational constant (G):	6.672×10^{-11} N m^{2} kg^{-2}
Mass of the Sun ($M_\odot$):	2×10^{30}kg
Radius of the Sun ($R_\odot$):	7×10^{8}m

SECTION A - Answer any SIX questions from this section

1.1) Briefly describe how a star's effective temperature can be estimated **either** by the use of Wein's Law **or** by the use of the Planck function. Summarise the strengths and weaknesses of the method you have chosen.

[7 marks]

1.2) In order for two protons to combine, they must overcome the Coulomb potential barrier. If the thermal and Coulomb energies are equated, then the temperature required to produce fusion for two protons is about 10^{10}K. However, the temperature at the core of the Sun is about 10^{7}K. With the aid of a sketch explain how it is possible for fusion to occur at this temperature.

[7 marks]

1.3) In the solar spectrum, the ratio of the strength of the calcium H line to that of the hydrogen Balmer Hα line is 400:1. Given that 95% of calcium in the Sun's photosphere can produce a calcium H line, but only 4.9×10^{-7} % of the hydrogen atoms can produce a Balmer absorption line, calculate the relative abundance of calcium to hydrogen in the Sun's photosphere.

[7 marks]

1.4) If a binary star system has a combined magnitude of +5 and one of the stars is twice as bright as the other (i.e. $F_a = 2F_b$), determine the magnitudes of the individual stars.

[7 marks]

1.5) Briefly describe the four main sources of opacity in stellar atmospheres.

[7 marks]

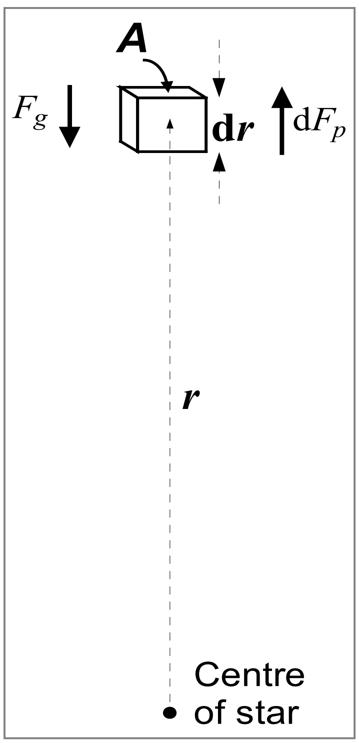
1.6) The core of an 8 $M_{\odot}$ star reaches the point in its evolution where the inner core consists mainly of ${}^{56}_{26}\mathrm{Fe}$. List the sequence of events which give rise to a type II supernova as the core of this star collapses.

[7 marks]

1.7) The apparent magnitude (m) of stars in an open star cluster (cluster *a*) are measured and plotted on an HR diagram. The main sequence stars on this plot are fitted to the main sequence of a HR plot of a nearby star cluster (cluster *b*) for which the absolute magnitude (M) is known. The best fit is obtained when the cluster *a* have to be shifted up by m-M = 6.3. What is the distance of cluster *a* in parsecs?

[7 marks]

1.8) By considering the forces acting on the small volume of gas indicated in the diagram, show that, under the conditions of hydrostatic equilibrium



$$\frac{\mathrm{d}P}{\mathrm{d}r} = -G\frac{M_r\,\rho}{r^2},$$

where M_r is the mass within a radius *r* and the other symbols have their usual meanings.

[7 marks]

SECTION B - Answer any TWO questions

2) Show that the total amount of gravitational energy released (E_G) in forming a star of radius R is

$$E_G = -\int_0^R \frac{GM_r}{r} \rho 4\pi r^2 \mathrm{d}r$$

where M_r is the mass within a radius r and the other symbols have their usual meanings.

[7 marks]

Given that the thermal energy per unit volume of an ideal gas containing n particles per unit volume is equal to $n\frac{3}{2}kT$, and that the equation of hydrostatic equilibrium is

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -G\frac{M_r \rho}{r^2},$$

show that

$$E_{thermal} = -\frac{1}{2}E_G$$

[14 marks]

Calculate the amount of gravitational energy released during the collapse of the Sun to its present radius. You may approximate the density of the Sun as being the Sun's mass divided by its volume.

[5 marks]

If we assume that the Sun's luminosity has been reasonably constant during its contraction on to the main sequence at $L_{\odot}=3.8\times10^{26}\,\mathrm{J\,s^{-1}}$, calculate the time taken for the Sun to condense on to the main sequence.

[4 marks]

3) Sketch a graph of the fraction of all hydrogen atoms which are ionised as a function of temperature for a temperature range of 5000K to 15000K. Explain why the strength of the hydrogen Balmer Hα absorption line is a maximum at ~ 10,000 K for main sequence stars.

[10 marks]

The ratio of the number of atoms in excited states of energies E_a and E_b in a gas at a temperature T may be calculated by the Bolzmann equation

$$\frac{N(E_b)}{N(E_a)} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

where g_a and g_b are the number of states associated with the energy levels E_a and E_b. The Saha equation which relates the ratio of the number of atoms in the i+1th state of ionisation to the ith state of ionisation can be written for the hydrogen atom as

$$\frac{N_{i+1}}{N_i} = \frac{2kT}{P_e} \frac{1}{2} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi/kT} \quad ,$$

where P_e is the electron pressure of a gas composed of hydrogen atoms which have an ionisation energy χ= 13.6 eV.

Determine the percentage of hydrogen atoms in Rigel's photosphere which can produce a Balmer absorption line, given that Rigel's effective surface temperature is 10,100 K, the electron pressure in the photosphere is 20 Nm^{-2}.

[20 marks]

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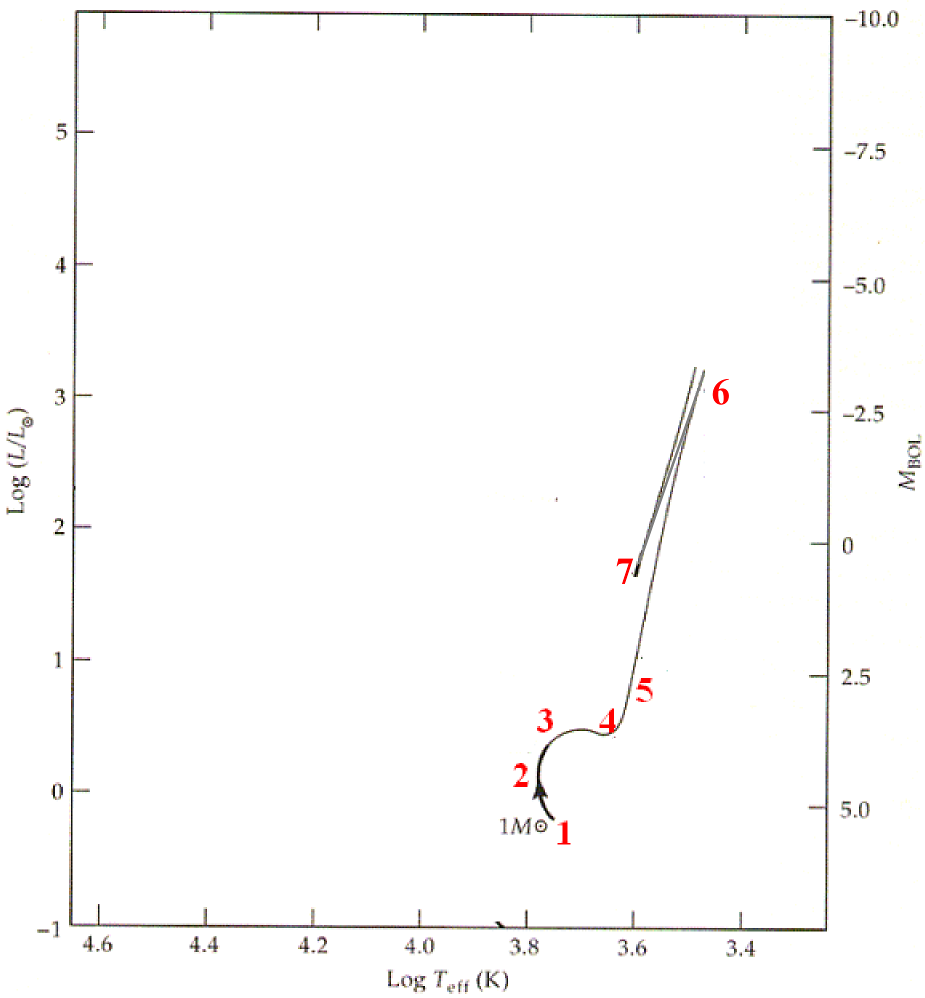
4) Describe what is meant by degenerate matter. Explain its importance with respect to the rate of fusion reactions at the core of stars which are evolving off the main sequence.

[6 marks]

The HR diagram below shows the evolution of a star of one solar mass from the start of hydrogen burning at point 1 on the diagram. Describe the physical processes which are responsible for the changes taking place between each of the numbered stages indicated on the diagram.



[13 marks]

At the end of the life of a one solar mass star, its outer layers can be thrown off by one or more "thermal pulses". Briefly explain the mechanism responsible for this.

[5 marks]

A star which has a mass of 15 $M_\odot$, evolves off the main sequence and for a period of time has the same surface temperature as the Sun. However, its luminosity is 10^5 times greater than that of the Sun. What is the radius of the star in units of $R_\odot$?

[6 marks]

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5) The equation of radiative transfer which relates the rate of change of intensity of a beam of light of intensity I_λ at a given wavelength with optical depth τ_λ can be written as

$$\frac{\mathrm{d}I_\lambda}{\mathrm{d}\tau_\lambda} = -I_\lambda + S_\lambda$$

where S_λ is the source strength at this wavelength and optical depth.

If a beam of light passes through a volume of gas *within* a black body, how would the input and output intensities compare for this volume?

[5 marks]

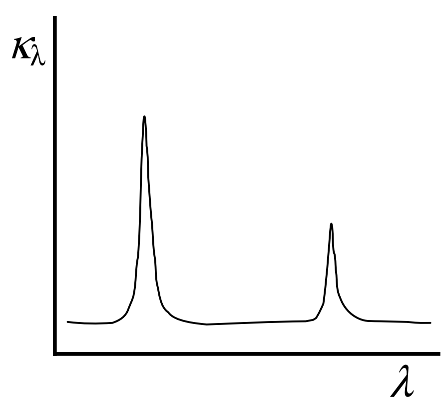
A beam of light passes through a volume of gas such that $I_\lambda(\tau = 0) = I_{o\lambda}$ and emerges after travelling through an optical depth τ_λ with an intensity I_λ .

Show that

$$I_\lambda(\tau) = -I_{o\lambda}e^{-\tau_\lambda} + S_\lambda\left(1 - e^{-\tau_\lambda}\right)$$

[15 marks]

A volume of hot gas has an opacity coefficient (κ_λ) which varies with wavelength as shown in the plot below.



The gas is not illuminated by external radiation. Explain how the equation above may be interpreted to predict the observed spectrum from the gas.

[2 marks]

Sketch what the spectrum would look like under the conditions where the gas is

i) optically thin [4 marks]

and

ii) optically thick. [4 marks]

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