King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP1490 Structure of Matter

Summer 2005

Time allowed: THREE Hours

Candidates should answer all SIX parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ε_0	=	8.854×10^{-12}	$\mathrm{F\ m^{-1}}$
Permeability of free space	μ_0	=	$4\,\pi \times 10^{-7}$	$\mathrm{H\ m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\ s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\ m^2\ kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_e	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_u	=	1.661×10^{-27}	kg
Proton rest mass	m_p	=	1.673×10^{-27}	kg
Neutron rest mass	m_n	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	J s
Boltzmann constant	k_B	=	1.381×10^{-23}	$\mathrm{J\ K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\ m^{-2}\ K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Avogadro constant	N_A	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\ m^{-2}}$
Acceleration due to gravity	g	=	9.810	$\mathrm{m\ s^{-2}}$

SECTION A – Answer all SIX parts of this section

1.1) Explain how Thomson was able to determine the charge to mass ratio e/m of the electron.

[7 marks]

1.2) A collimated beam of neutrons of wavelength 1.2 nm is emitted horizontally from a reactor. Determine how far the neutrons fall under gravity over a horizontal distance of 50 m.

[7 marks]

1.3) Use the Bohr model to show that the radius r of the hydrogen atom is given by

$$r = \frac{\varepsilon_0 h^2}{\pi m_e e^2}$$

[7 marks]

1.4) Define the mass defect and binding energy of a nucleus and state how they are related. Discuss the origin of the binding energy.

[7 marks]

1.5) Assume a human hair has a Young's modulus of 5.0×10^{8} $\mathrm{Nm^{-2}}$ and suffers damage at strains larger than 10%. Estimate the volume of hair required to construct a catapult capable of hurling a rock of mass 50 kg at a speed of 10 $\mathrm{ms^{-1}}$.

[7 marks]

1.6) State the Boltzmann distribution law and explain how it may be used to obtain the variation of atmospheric pressure with height, assuming the atmospheric temperature is independent of height.

[7 marks]

SECTION B – Answer TWO questions

2 a) X-rays are produced when a copper target is irradiated by electrons of energy 15 keV. Sketch a fully labelled graph of the intensity of the radiation as a function of X-ray wavelength. Indicate on the graph those features corresponding to the Bremsstrahlung and to the characteristic K_α, K_β and L_α emission peaks, which occur at 0.154, 0.139 and 1.33 nm, respectively. Explain the mechanisms which give rise to the Bremsstrahlung and to the characteristic emission peaks.

[15 marks]

Discuss how the X-ray emission spectrum would change if the energy of the electrons was increased to 20 keV.

[3 marks]

b) Moseley's Law relates the wavelength, λ_α, of the characteristic K_α emission peak of an element, to the atomic number Z of the element by the equation:

$$\frac{1}{\lambda_\alpha} = \frac{3E_0}{4hc}(Z-1)^2 ,$$

where E_0 = 13.6 eV is the binding energy of the lowest energy state of the hydrogen atom. The energy levels E_n in the Bohr model of a single-electron ion of nuclear charge $+Ze$ are given by

$$E_n = -\frac{Z^2 E_0}{n^2} .$$

Show how the Bohr model can be used to account for Moseley's equation.

[8 marks]

c) Use Moseley's equation to determine the atomic number of copper.

[4 marks]

3 a) The potential energy $V(r)$ between atoms can be descibed by the expression:

$$V(r) = \frac{A}{r^m} - \frac{B}{r^n}$$

where r is the separation between atoms, A and B are constants, and m and n are integers ($m > n$). Explain the physical origin of the two terms in the above equation.

Determine an expression for the force $F(r)$ and, hence, derive expressions for the constants A and B in terms of the equilibrium separation $r = a_0$ and the separation energy $\varepsilon = -V(a_0)$.

[12 marks]

b) Sketch a graph of stress versus strain for a ductile metal wire up to its fracture point. Label on the graph the proportionality point and the yield point.

Describe how the form of the graph can be related to the form of $F(r)$ for strains below the yield point

Describe the physical processes occuring in the metal for strains above the yield point.

[10 marks]

c) If there are $1/\, a_0^2$ bonds per unit area, $m = 12$ and $n = 6$, show that the Young's modulus E for the metal can be given by

$$E = \frac{72\varepsilon}{a_0^{\;3}}$$

[8 marks]

4 a) State the basic assumptions underlying the kinetic theory of gases. How does kinetic theory account for the pressure of a gas in a container? Explain why some of these asumptions are only valid for gases at low pressures.

[6 marks]

b) Explain what is meant by the mean free path length in a gas.

Show that the mean free path length λ of molecules of diameter d in an ideal gas is given by the expression

$$\lambda = \frac{1}{\pi n d^2} ,$$

stating any assumptions made. n is the number of gas molecules per unit volume.

Explain how this expression should be modified for an atom or molecule of diameter d_1 moving through a gas of molecules of diameter d_2.

[11 marks]

c) Diffusion may be described as the random walk of an atom or molecule as it collides with other molecules in a gas. Show that the root-mean-square distance r_{rms} travelled after a time t by an atom as it diffuses through a gas is given by

$$r_{\mathrm{rms}}{}^2 = \langle v \rangle \lambda t ,$$

where $\langle v \rangle$ is the mean speed of the atom.

[5 marks]

A small volume of radioactive gas is released into the air. Determine how long it will take before the radioactive atoms are detected a distance of 1 m from the point of release. The air is at room temperature and standard pressure. You may assume the diameter of the radioactive atoms and the air molecules both to be 0.3 nm. The mass of the radioactive atom is 222 m_{u}. Comment on your answer.

[8 marks]