King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the University counting towards the award of a degree. Examinations of the University are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/1490 THE STRUCTURE OF MATTER

JANUARY 2001

Time allowed: THREE HOURS

Candidates must answer any SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each section of the paper.

The approximate mark for each part of a question is indicated in the square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Constants **CP1490**

Electron charge, e	$= 1.602\times10^{-19}$ C
Electron mass, m_e	$= 9.109\times10^{-31}$ kg
Planck constant, h	$= 6.626\times10^{-34}$ J s
Speed of light in a vacuum, c	$= 2.998\times10^{8}$ m s^{-1}
Atomic mass unit, u	$= 1.662\times10^{-27}$ kg
Boltzmann constant, k	$= 1.381\times10^{-23}$ J K^{-1}
One Curie	$= 3.700\times10^{10}$ disintegrations a second
Half life of the radio isotope, ^{14}C	= 5730 years
Electrostatic constant, $1/4\pi\varepsilon_0$	$= 8.988\times10^{9}$ N m^{2} C^{-2}
Energy of an electron in the nth orbital of a hydrogen atom, E_n	$= -13.6/n^2$ eV

SECTION A - Answer any SIX questions from this section

1.1) Derive an expression for the mean free path of simple gas molecules. A room has a volume of 10 m^3. The room contains 2.5x10^{26} gas molecules - each of which has a diameter of 0.5 nm. Calculate the mean free path of the molecules in the room.

[7 marks]

1.2) A helicopter lifts a 500 kg mass using a nylon rope which has a natural length of 10 m and a cross-sectional area of 25 mm^2. Young's modulus for the rope is 2×10^9 N m^{-2}. Calculate the length of the rope under load and the energy required to stretch the rope before the mass can be lifted. You make take the acceleration due to gravity as 10 m s^{-2}.

[7 marks]

1.3) List the fundamental forces of nature. Place these in order of strength when acting on two protons when separated by 1×10^{-15} m. Draw a Feynman diagram which represents two protons repelling each other.

[7 marks]

1.4) Draw a labelled diagram showing the essential features of an atomic force microscope. State what each of your labelled components does.

[7 marks]

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1.5) The mass of a $^{112}_{48}Cd$ atom is measured using a mass spectrometer. The measured mass is slightly different from the value obtained by multiplying the atomic mass number by the atomic mass unit. The difference is greater than the experimental error. Give two reasons why is this so?

[7 marks]

1.6) The research reactor in Grenoble can produce a beam of neutrons which have a speed of 2×10^4 m s^{-1}. These neutrons are incident on a thin flat crystal at right angles to its face. They are diffracted by atomic planes which have a spacing of 0.4 nm. Calculate the deBroglie wavelength of the neutrons and determine the diffraction angle where the neutron beam will show a first maximum. Take the mass of a neutron as one atomic mass unit.

[7 marks]

1.7) Sketch a graph of the binding energy per nucleon against atomic mass number. Briefly explain the phenomena that give rise to its form.

[7 marks]

1.8) Sketch the form of the potential energy between two atoms as a function of displacement. With reference to your sketch, briefly explain why most solids expand when heated.

[7 marks]

SECTION B - Answer any TWO questions

2) How does the standard model of particle physics explain β^- decay?

[8 marks]

$^{60}_{27}\text{Co}$ has an atomic mass of 59.93 atomic mass units and is made in a nuclear reactor. It is radioactive with a half-life of 1925 days. A sample of this isotope gives an initial activity of 10^4 Curies. Calculate the mass of the sample.

[12 marks]

Due to a breach in reactor safety protocols, an unfortunate radiation worker ingests some of this isotope and an equal mass of a second isotope, $^{57}_{27}\text{Co}$, which has a half life of 272 days. Both isotopes were readily taken up by the worker's bones. On the day he died, his bones were analysed and found to contain $^{60}_{27}\text{Co}$ and $^{57}_{27}\text{Co}$ in the ratio 2.23:1 by mass. How many days had elapsed since the accident?

[10 marks]

3) In the Bohr model of the hydrogen atom, the electron orbits have quantised angular momenta equal to $n\hbar$, where n is an integer and $\hbar$ is the Planck constant divided by 2π.
Show that the radius (r_n) of the n^{th} orbit in a hydrogen atom is given by

$$r_n = \frac{4\pi\,\varepsilon_0 \hbar^2 n^2}{me^2} \quad .$$

[9 marks]

By considering the energies that an electron has in the hydrogen atom, show that the energy of the n^{th} state is

$$E_n = -\frac{e^4 m}{2(4\pi\varepsilon_0)^2 \hbar^2} \frac{1}{n^2} \quad .$$

[9 marks]

The Rosette nebula close to the constellation Orion consists of a large cloud of ionised hydrogen and helium. Electrons are recombining with these ions and are emitting radiation. What wavelength is produced when a free electron combines with a doubly ionised *helium* ion and makes a transition from the n = 3 to the n = 2 orbital?

[12 marks]

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4) Electrons can be considered as particles or waves. Briefly describe **either** an experiment which demonstrates *particle* behaviour **or** an experiment which demonstrates *wave* behaviour.
[6 marks]

Draw a labelled diagram of Thomson's apparatus for measuring the charge-to-mass ratio of the electron. Explain how the value of the charge to mass ratio may be obtained from its use.
[14 marks]

The electron gun in Thomson's apparatus is replaced by a high-strength radioactive source, which emits α particles with a velocity (v_α) of 5×10^6 m s^{-1}. In the absence of electric and magnetic deflection fields, a bright spot is seen on the centre of the fluorescent screen. A uniform magnetic field (B) of 0.2 tesla is applied at right angles to the direction of motion of the α particles for the last 10 cm of their path to the screen. If the distance for which the electrons travel under the influence of the magnetic field is p, show that the deflection of the of a beam would be described by

$$s_\alpha = \frac{B\,e}{m_\alpha v_\alpha}(p)^2$$

and thence determine the magnitude of the deflection.

[10 marks]

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5) Argon atoms are cooled to the point where they form a liquid. The potential, $V(r)$, experienced by an argon atom separated from *one* other argon atom (by a centre to centre distance r) can be described by the Lennard-Jones 6-12 potential

$$V(r) = \xi\left[\left(\frac{a_0}{r}\right)^{12} - 2\left(\frac{a_0}{r}\right)^{6}\right],$$

where ξ is the binding energy of the two atoms, a_0 is their mean separation.

Sketch the form of this potential, and indicate how at low temperatures, the vibration of the atoms approximate simple harmonic motion. Using this concept, show that the frequency of vibration (ν) of argon in a liquid state can be expressed as

$$\nu = \frac{1}{2\pi}\sqrt{\frac{n\xi\, 72}{3\, a_0^2 m}}$$

where m is the mass of an argon atom and n is the number of nearest neighbours.

[12 marks]

Now consider an ion in an ionic crystal. The potential experienced by an ion in this crystal due to all other ions is given by

$$V(r) = \frac{1.75\, e^2}{4\pi\varepsilon_o}\left(\frac{a_o^9}{10\, r^{10}} - \frac{1}{r}\right)$$

Derive an expression for frequency of vibration of ions experiencing this potential.

[10 marks]

The equilibrium separation for argon atoms in the fluid and ions in the crystal are about the same at $a_0 \approx 4\times10^{-10}$m. The masses of the ions and argon atoms are also about equal. The binding energy for an argon atom in the liquid is 0.011 eV. How would the frequency of vibration compare between the ionic crystal and the liquid argon?

[8 marks]