

# King's College London

UNIVERSITY OF LONDON

**This paper is part of an examination of the University counting towards the award of a degree. Examinations of the University are governed by the Senate Regulations.**

**B.Sc. EXAMINATION**

**CP/1490 THE STRUCTURE OF MATTER**

**JANUARY 2000**

**Time allowed: THREE HOURS**

**Candidates must answer any SIX parts of SECTION A,  
and TWO questions from SECTION B.**

**The approximate mark for each part of a question is indicated in the square brackets.**

**Separate answer books must be used for each section of the paper.**

**TURN OVER WHEN INSTRUCTED**

## Constants

Electron charge ( $e$ ):	$1.602 \times 10^{-19} \text{ C}$
Electron mass ( $m_e$ ):	$9.109 \times 10^{-31} \text{ kg}$
The Planck constant ( $h$ ):	$6.626 \times 10^{-34} \text{ J s}$
Speed of light in a vacuum ( $c$ ):	$2.998 \times 10^8 \text{ ms}^{-1}$
Atomic mass unit (u):	$1.662 \times 10^{-27} \text{ kg}$ or $931.5 \text{ MeV}/c^2$
The Boltzmann constant ( $k$ ):	$1.381 \times 10^{-23} \text{ J K}^{-1}$
Natural log of 2 ( $\ln(2)$ ):	0.693
Half life of the radio isotope $^{14}\text{C}$ :	5730 years
Mass of a Hydrogen atom ( $m_{\text{H}}$ ):	1.00783 u
Electrostatic constant ( $\frac{1}{4\pi\epsilon_0}$ ):	$8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

## SECTION A - Answer any SIX questions from this section

- 1.1) In the Bohr model of the hydrogen atom, the electron orbits have quantised angular momentum equal to  $nh$ , where  $n$  is an integer and  $h$  is Planck's constant divided by  $2\pi$ . Show that the radius ( $r_n$ ) of the  $n$ th orbit in a hydrogen atom is given by

$$r_n = \frac{4\pi \epsilon_0 \hbar^2 n^2}{me^2}$$

[7 marks]

- 1.2) The kinetic energy  $E$  per atom of an ideal gas atom can be calculated from  $E = \frac{3}{2}kT$ , where  $T$  is the temperature in degrees Kelvin and  $k$  is the Boltzmann constant. An energy of 16 eV is required to ionise argon atoms. Briefly explain why a small percentage of argon atoms can be ionised by collisions at room temperature.

[7 marks]

- 1.3) Explain the function of the moderator in a fission based nuclear reactor. Calculate the kinetic energy transferred when a neutron which is travelling at  $1 \times 10^8 \text{ m s}^{-1}$  gives up most of its energy in collisions in a nuclear reactor. You may approximate the mass of neutron as being one atomic mass unit.

[7 marks]

- 1.4) What is meant by the term "Q value" when applied to nuclear reactions? What is the significance of its sign?

[7 marks]

- 1.5) In the Thomson model of the atom, electrons were thought to be embedded in a cloud of positive charge. Briefly discuss one piece of experimental evidence which cannot be reconciled with this model.

[7 marks]

- 1.6) A fragment of wood from a site believed to be from an early Roman settlement is analysed by accelerator mass spectroscopy. The analysis shows the ratio of the number of  $^{14}\text{C}$  atoms to the number of  $^{12}\text{C}$  atoms to be  $7.82 \times 10^{-13}$ . Determine the age of the wood fragment.

[7 marks]

- 1.7) Sketch a graph of the relationship between atomic number and atomic mass number for stable nuclei. Briefly explain the form of the graph.

[7 marks]

- 1.8) The atoms in a  $\text{CO}_2$  molecule are arranged in a straight line with the carbon atom in the middle. Describe the origin and number of absorption peaks which occur in the infrared spectrum of this molecule.

[7 marks]

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## SECTION B - Answer any TWO questions

- 2) A small lump of uranium 238 ( $^{238}\text{U}$ ) emits  $\alpha$  particles with an energy of 4.27 MeV. This  $\alpha$  particle source is introduced into the entrance of a cloud chamber. The cloud chamber has a magnetic field of 2 Tesla applied to it with the field direction normal to the direction of motion of the  $\alpha$  particles. The  $\alpha$  particles have a mass of  $6.65 \times 10^{-27}$  kg.

Sketch the path of the  $\alpha$  particles in the cloud chamber.

[4 marks]

Calculate the radius of the  $\alpha$  particle trajectories in the cloud chamber.

[8 marks]

In one hour the  $^{238}\text{U}$  source emits  $10^6$   $\alpha$  particles. Calculate the mass of  $^{234}\text{Th}$  created in this time.

[9 marks]

Calculate the mass of the lump of uranium. Justify any approximations made.

[9 marks]

Mass of  $^{238}\text{U}$  atom = 238.05078 u

Mass of  $^4\text{He}$  atom = 4.00260 u

The half life of  $^{238}\text{U}$  is  $4.47 \times 10^9$  years

- 3) Describe with the aid of a diagram how a scanning electron microscope can be used to determine the elemental composition of specimens.

[11 marks]

A sample is placed in the scanning electron microscope for analysis. It is known to contain copper. The x-ray energy spectrum shows copper  $K\alpha$ ,  $K\beta$ , and  $L\alpha$  emission peaks at 8040, 8904, and 930eV. Draw an energy level diagram and indicate the transitions that give rise to these peaks.

[7 marks]

The x-ray detector in the above system has an entrance window which is coated with a  $0.2 \mu\text{m}$  thick layer of gold through which x-rays must pass before they are detected. The x-ray photon emission is recorded for a set period of time during which 8410 x-ray photons are detected due to the copper  $K\alpha$  radiation while 200 x-ray photons are recorded due to the copper  $L\alpha$  radiation. Calculate the ratio of the numbers of  $K\alpha$  and  $L\alpha$  x-rays emitted from the sample.

Gold has a linear x-ray absorption coefficient of  $11.04 \mu\text{m}^{-1}$  for copper  $L\alpha$  radiation and  $0.394 \mu\text{m}^{-1}$  for copper  $K\alpha$  radiation.

[12 marks]

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- 4) A metal wire is placed under increasing stress up to a level where a permanent deformation is induced in the wire. Sketch a graph of stress versus strain for the wire and label it appropriately. Give a brief description of the physical processes that give rise to its form. [10 marks]

Guitarists frequently break the thinnest “E” string on electric guitars. Figure 1 shows a schematic diagram of the strings and frets on a guitar. The E string is undergoing a strain while being played. This string has an un-strained length of 0.8 m and is made from a metal with a Young’s modulus of  $4.0 \times 10^{10} \text{ N/m}^2$ . And breaking stress of  $2 \times 10^9 \text{ N/m}^2$ . Calculate the displacement,  $d$ , of the string at the mid-string play position which will break the string. For the purposes of the calculation you may assume that the string is un-strained before it is displaced.

[10 marks]

In practice, tuning pre-strains the string by 3%. How does this alter your calculated value of breaking displacement?

[10 marks]

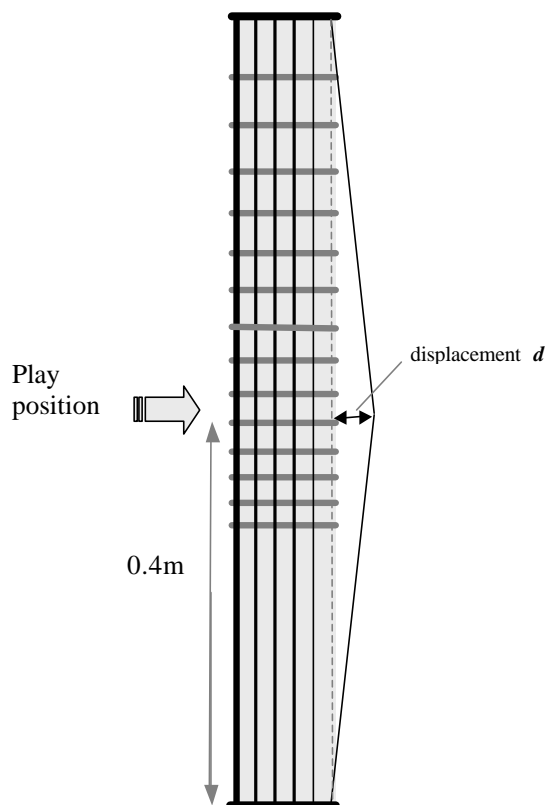


Figure 1

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- 5) Sketch a graph of the inter-atomic potential for two atoms in a metal lattice and use this to explain why the metal expands when heated.

[8 marks]

At low temperature, the oscillations of an atom in a solid about its equilibrium position are small and the motion of the atoms about the equilibrium approximate to simple harmonic motion. Using this approximation, show that the frequency of oscillation  $\mathbf{n}$  is given by

$$\mathbf{n} = \frac{1}{2\mathbf{p}} \sqrt{\frac{d^2V/dr^2}{m}},$$

where  $r$  is the displacement about the equilibrium position,  $V$  is the potential experienced by the atom and  $m$  is its mass.

[8 marks]

The potential of an ion in a metal-halide crystal can be written as

$$V(r) = \frac{1.75e^2}{4\mathbf{p}e_0} \left( \frac{a^{11}}{12r^{12}} - \frac{1}{r} \right),$$

where  $a$  is the distance between ions at equilibrium. Show that the frequency of oscillation is given by

$$\mathbf{n} = \frac{1}{2\mathbf{p}} \sqrt{\frac{1.75e^2}{4\mathbf{p}e_0} \left( \frac{13a^{11}}{r^{14}} - \frac{2}{r^3} \right)}.$$

[8 marks]

Positive ions in this crystal have a mass of  $3.8 \times 10^{-26}$  kg and have an equilibrium separation of  $3.0 \times 10^{-10}$  m. Calculate the frequency of oscillation of these ions at low temperature.

[6 marks]