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KEELE UNIVERSITY

EXAMINATIONS, 2012/13

Level III

Monday 29th April 2013, 09:30 – 11:30

PHYSICS/ASTROPHYSICS

PHY-30029

QUANTUM PHYSICS II

Candidates should attempt to answer THREE questions.

A sheet of useful information can be found on page 7.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. A sample of cold $^{12}C^{16}O$ molecules is exposed to a beam of neutrons with typical energies 0.01 eV. The spring constant for the C–O bond is $k = 1900\ \mathrm{N\,m^{-1}}$.

The ground state of a simple harmonic oscillator with mass m is

$$\psi_0(x) = \left(\frac{1}{a\sqrt{\pi}}\right)^{1/2} e^{-x^2/2a^2}.$$

where $a = \sqrt{\hbar/m\omega}$ and $\omega = \sqrt{k/m}$ and k is the spring constant.

The energy levels of a simple harmonic oscillator are $E_n = (n+\frac{1}{2})\hbar\omega$.

(a) Show that the energy of the neutrons is much less than the energy for vibration modes in the CO molecules. [20]

(b) Show that for an instantaneous change in the mass from m to m' the probability that the oscillator remains in the ground state is $|c_0|^2 = 2/\left((m'/m)^{\frac{1}{4}} + (m/m')^{\frac{1}{4}}\right)$. [40]

(c) Calculate the probability for a $^{12}C^{16}O$ molecule to be observed in the ground state following neutron capture to become $^{13}C^{16}O$. State clearly any approximations you have made. [15]

(d) Infrared radiation due to vibrational transitions $v = 1 \rightarrow 0$ is observed from the cold CO gas, but only when it is exposed to the neutron beam. Explain this observation and state where the energy for this radiation comes from. [25]

You can use the following integral without proof in your answer.

$$\int_{-\infty}^{\infty} e^{-cx^2} dx = \sqrt{\frac{\pi}{c}}$$

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2. Consider an electron in the state α_y. (The Pauli spin matrices given on page 7 of this paper.)

   (a) Write α_y as a linear superposition of α_z and β_z and hence show that the probability of observing this electron in the spin-down state on the z-axis is $\frac{1}{2}$. [20]

   (b) The Hamiltonian for the interaction of a localised electron with a magnetic field of strength B aligned with the z-axis is

   $$\hat{H} = \frac{egB}{2m_e}\hat{S}_z.$$

   For an electron in the state α_y at time $t = 0$ show that $\langle S_y \rangle = \frac{\hbar}{2}\cos(2\omega t)$, where $\omega = \frac{egB}{4m_e}$. [50]

   (c) Explain why the ability to manipulate electron spins may make it possible to build a quantum computer that can solve problems beyond the capabilities of digital computers. [30]

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3. The rotational energy levels of a rigid linear molecule are given

$$E_J = B\,J(J+1),$$

where $J = 0, 1, 2, 3, \ldots$ is the rotational quantum number and B is a constant.

Laser light scattered from C_2 gas at room temperature shows a series of several emission lines offset from the frequency of the laser by 0.329 THz, 0.768 THz, 1.207 THz, etc.

(a) Explain the origin of these emission lines. [20]

(b) Explain why the emission lines corresponding to odd J values are missing. [25]

(c) What is the value of B for C_2 in electron volts (eV)? [10]

(d) The fraction of molecules with rotational quantum number J at temperature T is $\eta = (2J+1)e^{-E_J/k_BT}$. Calculate the value of J for the most populated rotational energy level of C_2 gas at room temperature. [25]

(e) Why is a Raman spectrum with several emission lines unlikely to be due to vibrational Raman scattering? [20]

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4. Two ions are put in an entangled state

$$|\psi\rangle = \frac{(1+i)}{2\sqrt{2}}\left[|\uparrow\uparrow\rangle + i|\downarrow\downarrow\rangle - |\uparrow\downarrow\rangle + i|\downarrow\uparrow\rangle\right]$$

A laser pulse is then used to detect whether the ions are in the same state, i.e., either $|\uparrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle$. The experiment is repeated N times and the number of pairs of ions in the same state, N_{same}, is recorded.

(a) Explain what is meant by an entangled state and the implications for measurements performed on the system. [15]

(b) Use the operator $\hat{N}_{\mathrm{same}} = N\left[|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|\right]$ to calculate the value of $\langle N_{\mathrm{same}}\rangle$. [25]

(c) The experiment is repeated for pairs of ions in different entangled states and a quantity B is calculated from the resulting values of N_{same}. The observed value is $B_{\mathrm{obs}} = 2.25 \pm 0.03$.

i. Bell's inequality for this experiment is $B \leq 2$. Explain what is meant by this statement and why this inequality is violated in this experiment. [30]

ii. Almost all ion pairs produced in this experiment can be measured. Why is this important for experimental tests of Bell's inequality? [20]

iii. Suggest one reason why the observed value of B may be slightly lower than the value predicted using the value of $\langle N_{\mathrm{same}}\rangle$. [10]

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5. (a) Derive an approximate expression for the critical temperat for the formation of a Bose-Einstein condensate, T_c, for a collection of N particles of mass m in a volume V. Explain your method clearly. [20]

(b) A cloud of 500,000 ^{23}Na atoms is trapped in a region with a diameter of 0.02 mm.

i. Estimate the value of T_c for these atoms. [10]

ii. Estimate the size of the cloud at temperatures just above and just below T_c 0.01 s after the magneto-optical trap is turned off. [2×15]

(c) Outline the operating principles of a magneto-optical trap. [40]

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Pauli spin matrices

Operator	Eigenvalue	Eigenvector
$\hat{S}_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\frac{1}{2}\hbar$	$\alpha_x = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
	$-\frac{1}{2}\hbar$	$\beta_x = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
$\hat{S}_y = \frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	$\frac{1}{2}\hbar$	$\alpha_y = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$
	$-\frac{1}{2}\hbar$	$\beta_y = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$
$\hat{S}_z = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\frac{1}{2}\hbar$	$\alpha_z = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
	$-\frac{1}{2}\hbar$	$\beta_z = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$