The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2012/13

Level III

Tuesday 30th April 2013, 09.30-11.30

PHYSICS/ASTROPHYSICS

PHY-30025

LIFE IN THE UNIVERSE

Candidates should attempt to answer THREE questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. Consider the formation of the amino-acid glycine ($O{=}CHCH_2N$… within an ice particle in space, from the subsequent reactions of formaldehyde ($O{=}CH_2$) with methanol (CH_3OH) and ammonia (NH_3). Some of the relevant bond energies are tabulated here:

bond	H–H	C–H	N–H	C–C	C–N
bond energy (kJ mol^{-1})	436	414	389	347	305

(a) Sketch, and describe in more detail, how the above reactions proceed. [20]

(b) Calculate the net energy of the chain of reactions. [10]

(c) Give two reasons for why the formation of glycine is unlikely to happen spontaneously. [10]

(d) Calculate the wavelength of a photon that could help initiate the reactions; comment on your answer. [20]

(e) Calculate the gas temperature required for electrons to have sufficient energy to help initiate the reactions; comment on your answer. [20]

(f) Discuss the relevance of the above discussions for the emergence of life on Earth. [20]

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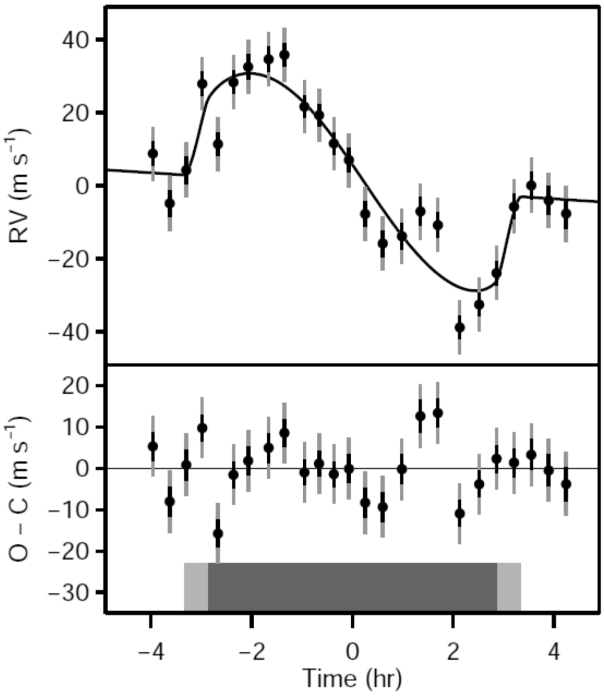
2. The figure below shows radial velocity (RV) measurements of a s[illegible] of mass $M_\star = 1.2\ \mathrm{M}_\odot$ and radius $R_\star = 1.6\ \mathrm{R}_\odot$ during a planet transit; the line is a model, and the lower panel shows the difference between the observed (O) and calculated (C) velocity.



(a) Explain the observed radial velocity variations. [20]

(b) If the period of the full radial velocity modulation is 20 days, determine the distance between the planet and the star. [20]

(c) If the radial velocity of the star varies between $-80\ \mathrm{m\ s^{-1}}$ and $+80\ \mathrm{m\ s^{-1}}$, determine the mass of the planet. [20]

(d) Estimate the radius of the planet; account for inclination. [20]

(e) Describe how additional observations can help to improve the determinations of the planet mass and radius. [20]

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3. (a) Calculate the equilibrium temperature of the Earth, assum an albedo of 0.3 and an emission efficiency of 0.6. [20]

(b) The Earth's atmosphere has a total mass $M = 5 \times 10^{18}$ kg and a specific heat capacity of $c_P = 1$ kJ kg^{-1} K^{-1}. Estimate how long it takes for the Earth's atmosphere to cool by 1 K. [20]

(c) Consider the thermal diffusion equation

$$\frac{dq}{dt} = -k\,\frac{dT}{dx},$$

where dq/dt is the rate of heat flow per unit area, k is the thermal conductivity and dT/dx is the temperature gradient. The thermal diffusivity

$$\alpha = \frac{k}{\rho c_P},$$

where ρ is the density and c_P is the specific heat capacity.

i. Show that the velocity at which the temperature gradient is annihilated

$$\frac{dx}{dt} = \frac{\alpha}{L},$$

where L is a length scale. [20]

ii. In Earth's atmosphere, $\alpha = 1.9 \times 10^{-5}$ m^2 s^{-1} and the mean free path $\delta = 68$ nm. Hence argue that $dx/dt < \alpha/\delta$. [10]

(d) Assuming a balance between the thermal pressure gradient and the ram pressure, estimate the typical (maximum) wind speed one may expect. [20]

(e) Considering the equatorial rotation speed at Earth's surface, comment on the ability of diffusion and wind to reduce day–night temperature variations. [10]

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4. (a) Explain how the Earth's surface is protected against harmful radiation from the Sun. [20]

(b) Explain why the cosmic ray flux at Earth's surface is at its maximum when the Sun is least active. [20]

(c) Explain how Earth's exposure to harmful radiation and cosmic rays has changed since the formation of the Solar System. [20]

(d) Consider the vertical motion of the Solar System in and out of the Milky Way disc mid-plane, described by

$$\frac{d^2 z}{dt^2} = -\frac{d\Phi}{dz},$$

where Φ is the gravitational potential – which can be found from the Poisson equation

$$\frac{d^2\Phi}{dz^2} = 4\pi G\rho(z),$$

where ρ is the density.

i. By Taylor expansion of Φ, show that the equation of motion takes the form

$$\frac{d^2 z}{dt^2} = -\omega^2 z.$$ [20]

ii. For $\rho \sim 1\ \mathrm{M_\odot\ pc^{-3}}$, calculate the period of motion and comment on the possible significance for life on Earth. [20]

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5. Consider building a spacecraft which is driven by means of a so sail made of reflecting foil of surface mass density $\Sigma = 1$ g m^{-2}.

   (a) Show that the acceleration is independent of the surface area of the sail as long as the sail dominates the total mass. [10]

   (b) The spacecraft starts at rest at 1 au distance from the Sun. It then falls towards the Sun, passing it at a distance of 10 solar radii at which time the sail unfolds. Use energy arguments to show that it would take $< 13\,000$ yr to travel to a star at a distance of 10 light years. [30]

   (c) Assume in the following that once every million yr, each colony will send an identical spacecraft on a voyage to another star at a distance of 10 light years.

      i. Set up a differential equation which describes the growth of the number of colonies with time, $N(t)$, and hence derive the following expression:

      $$N(t) \;=\; N_0 \;\exp(\alpha\; t). \qquad [10]$$

      ii. Hence determine the number of colonies after $t = 10$ million and $t = 100$ million yr. [10]

      iii. Considering the Milky Way is pretty much a flat disc, with a diameter of 100 000 light years, estimate the time it takes for the colonization of the entire Milky Way. [30]

      iv. If only 10% of voyages are successful, 10% of settlements are successful, and 10% of colonies continue colonization, estimate the number of colonies after $t = 1$ billion yr. [10]