The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2012/13

Level III

Friday 11$^{\text{th}}$ January 2013, 13.00-15.00

PHYSICS/ASTROPHYSICS

PHY-30023

PARTICLES, ACCELERATORS AND REACTOR PHYSICS

Candidates should attempt to answer THREE questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. (a) Some of the quantum numbers of hadronic states include: Q the electromagnetic charge (in units of e); B for baryon number; S for strangeness; C for charm; $\widetilde{B}$ for the bottom quantum number; and T for the top quantum number.

Infer the quark contents implied by each of the following sets of quantum numbers for possible hadronic states, and thus determine whether or not each is compatible with the quark model.

i. $(Q, B, S, C, \widetilde{B}, T) = (2, 1, 0, 1, 0, 0)$ [10]

ii. $(Q, B, S, C, \widetilde{B}, T) = (-1, 1, 0, 1, -1, 0)$ [10]

(b) Briefly explain what is meant by strong *isospin* and *isospin multiplet*. State the quantum numbers I and I_3 for all quarks in the Standard Model. Give the names, quark contents, and isospin quantum numbers of all members of the lowest-mass isospin multiplet of baryons. [20]

(c) Write down all of the possible quark contents for baryons with $C = 1$, $\widetilde{B} = 0$ and $T = 0$. Plot these combinations on a graph of hypercharge, Y, versus I_3. Identify the isospin multiplets on this diagram. [35]

(d) The baryons Σ_c^{*0}, Ξ_c^{*0} and Ω_c^{*0} all have $C = 1$, $\widetilde{B} = 0$, $T = 0$, and spin-parity $J^P = \frac{3}{2}^+$. The Σ_c^{*0} has a rest mass of 2518 MeV$/c^2$; the Ξ_c^{*0} has rest mass 2646 MeV$/c^2$; and the Ω_c^{*0} has rest mass 2766 MeV$/c^2$. For each of these particles determine, with justification,

i. the quark content; [15]

ii. the electromagnetic charges of all other baryons in the same isospin multiplet from part (c). [10]

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2. (a) The Klein-Gordon wave equation in quantum mechanics is

$$-\hbar^2 \frac{\partial^2 \phi}{\partial t^2} \;=\; -\,\hbar^2 c^2\, \nabla^2 \phi \,+\, m_0^2 c^4\, \phi \;.$$

i. Assuming a plane-wave solution of the form

$$\phi(\mathbf{r}, t) \propto \exp\left[i(k_x x + k_y y + k_z z - \omega t)\right] \;,$$

find the dispersion relation (ω versus k) for free particles. What is the classical equivalent of this relation? [20]

ii. Show that a *static and spherically symmetric* solution of the equation is

$$\phi(r) \;=\; -\,\frac{g^2}{4\pi\, r}\, e^{-r/R} \qquad \text{with} \qquad R \equiv \hbar/(m_0 c) \;.$$

Discuss the physical significance of ϕ in this case, and of the quantity $-g^2/4\pi$. [25]

iii. Show how the constant $R = \hbar/(m_0 c)$ is related quantitatively to the maximum range of a force mediated by a virtual particle. Estimate the range of the electromagnetic force and the range of the weak force. [25]

(b) Define each of the following terms and explain their physical significance:

i. Cabibbo angle; and [15]

ii. Weinberg angle. [15]

<u>USEFUL INFORMATION</u>

For a spherically symmetric function $f(r)$,

$$\nabla^2 f(r) \;=\; \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{df}{dr}\right) \;=\; \frac{1}{r}\frac{d^2}{dr^2}\left[r\, f(r)\right] \;.$$

The $W^{\pm}$ rest mass is $m_W c^2 = 80.40$ GeV.

In convenient units, $\hbar c = 1.973 \times 10^{-16}$ GeV m.

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3. (a) A particle with charge q, rest mass m_0, kinetic energy K and total energy E moves on a path with radius of curvature r in a plane perpendicular to a uniform magnetic field of strength B.

   i. Show that
$$r^2 = \frac{E^2 - m_0^2 c^4}{q^2\, c^2\, B^2}\,. \qquad [10]$$

   ii. Show that the frequency of a full circular orbit in the field is given by
$$f_{\rm orb} = \frac{qBc^2}{2\pi\,(K + m_0 c^2)}\,. \qquad [10]$$

(b) Protons are injected with $K = 1$ MeV into a cyclotron operating with $B = 1.4$ T and a constant frequency $f_{\rm AC} = 21.35$ MHz for the accelerating voltage. The protons are extracted at a radius $R = 75$ cm. Calculate the kinetic energy of the protons when they are extracted, and determine the initial and final values of the ratio $f_{\rm orb}/f_{\rm AC}$. [15]

(c) Briefly explain the fundamental limitation of cyclotrons, and state how this limitation is dealt with in AVF cyclotrons, in synchrocyclotrons, and in synchrotrons. [30]

(d) Protons are injected with $K = 450$ GeV into a synchrotron with physical radius $R = 4.3$ km, where the magnetic field strength in the bending magnets varies from $B = 0.54$ T to $B = 8.4$ T.

   i. Calculate the final energy of the protons. [10]

   ii. Calculate the initial and final revolution frequencies of the protons. [10]

   iii. If the accelerating voltage per revolution is 490 kV, then how long does it take the protons to be accelerated to their final energy? [15]

<u>*USEFUL INFORMATION*</u>: *Proton rest mass is* $m_p c^2 = 938.3$ *MeV.*

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4. (a) Two particles with rest masses m_A, m_B, momenta $\mathbf{p}_A$, $\mathbf{p}_B$ and total energies E_A, E_B collide at a crossing angle θ. Show that the total energy in the centre-of-mass frame is

$$E_{\mathrm{CM}}^2 \; = \; m_A^2 c^4 + m_B^2 c^4 + 2E_A E_B + 2|\mathbf{p}_A||\mathbf{p}_B|c^2 \cos\theta \; . \qquad [25]$$

(b) The HERA collider, which was operational until 2007, collided electron beams of energy 30 GeV with proton beams of energy 820 GeV. Evaluate E_{CM} for a head-on collision of these beams; and calculate the energy that an electron incident on a stationary proton would need in order to achieve the same E_{CM} in a fixed-target accelerator. [25]

(c) Real Z^0 bosons were first produced at CERN in collisions between proton and anti-proton beams at centre-of-mass energies $E_{\mathrm{CM}} = 540$ GeV. Given that the rest-mass energy of the Z^0 is of order $\sim$100 GeV, explain why such large E_{CM} are required to produce these bosons in $p\,\overline{p}$ collisions. [20]

(d) A Z^0 boson in motion decays into an electron-positron pair. The electron moves away with energy $E_1 = 51.7$ GeV, at an angle $\phi_1 = 60°$ to the original trajectory of the Z^0. The positron moves away at an angle $\phi_2 = -80°$ to the original Z^0 trajectory. Calculate:

i. the energy of the positron; [10]

ii. the rest-mass energy of the Z^0; [10]

iii. the kinetic energy of the Z^0 before it decayed. [10]

USEFUL INFORMATION

The proton rest mass is $m_p c^2 = 938.3$ *MeV and the electron rest mass is* $m_e c^2 = 0.5110$ *MeV.*

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5\. (a) Explain what is meant by the *neutron multiplication factor*, k, in a thermal fission reactor. State the *four-factor formula* for k, and discuss physically all of the terms that contribute to this formula. In what sense does k differ in reality from the value given by the four-factor formula? [55]

(b) The steady-state neutron flux in a homogeneous reactor medium satisfies the equation

$$\boldsymbol{\nabla}^2 \Phi(\mathbf{r}) \;=\; -\,\frac{(k_\infty - 1)}{L_{\rm tot}^2}\,\Phi(\mathbf{r})\ ,$$

where k_∞ is the neutron multiplication factor given by the four-factor formula and $L_{\rm tot}$ is an effective diffusion length.

i. Show that the general solution to this equation for a *spherical* reactor is

$$\Phi(r) \;=\; \frac{a}{r}\,\sin\left(B\,r\right) + \frac{b}{r}\,\cos\left(B\,r\right)\ ,$$

where

$$B \;=\; \frac{\sqrt{k_\infty - 1}}{L_{\rm tot}}$$

and a and b are constants of integration. [15]

ii. Apply appropriate boundary conditions on the formula for $\Phi(r)$ given in part (i), to derive an expression for the volume of the reactor in terms of k_∞ and $L_{\rm tot}$. [20]

iii. A particular spherical reactor operates at criticality while suffering leakage of 3.3% of its neutrons. If $L_{\rm tot} = 0.3$ m, then calculate the radius of the reactor. [10]

<u>*USEFUL INFORMATION*</u>

For a spherically symmetric function $f(r)$,

$$\boldsymbol{\nabla}^2 f(r) \;=\; \frac{1}{r^2}\,\frac{d}{dr}\left(r^2\,\frac{df}{dr}\right) \;=\; \frac{1}{r}\,\frac{d^2}{dr^2}\big[r\,f(r)\big]\ .$$