The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2012/13

Level III

Wednesday 24th April 2013, 13.00-15.00

PHYSICS/ASTROPHYSICS

PHY-30003

THE PHYSICS OF COMPACT OBJECTS

Candidates should attempt to answer THREE questions.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

1. (a) Explain what is meant by electron degeneracy and outline
this leads to the phenomenon of *degeneracy pressure.* [20]

(b) If the density of eigenstates of momentum p, is given by $g(p) = 8\pi p^2/h^3$, then show that the Fermi momentum is given by

$$p_F = (3h^3/8\pi)^{1/3}\, n^{1/3}\,,$$

where n is the fermion number density. [20]

(c) The central parts of the planet Jupiter are thought to have a density of $3900\,\mathrm{kg\,m^{-3}}$, a temperature of $13300\,\mathrm{K}$ and a gas pressure of $3.8 \times 10^{12}\,\mathrm{Pa}$. Assume that the core composition is a mixture of completely ionised silicon and oxygen.

i. Show that electrons in Jupiter's core are degenerate and that this largely accounts for the gas pressure. [20]

ii. Further out in Jupiter's envelope, the density falls and electrons and ions begin to recombine. On the same plot, *with the same scale*, sketch the electron occupation index as a function of electron energy for the gas in the core and for the gas in a region further out from the centre. [10]

iii. Explain, why the thermal and electrical conductivity of the gas in the core is high. [10]

iv. Jupiter is cooling down. With the aid of a sketch of radius versus time, describe qualitatively how and why the radius of Jupiter has changed and will change as it gets older, and explain why its core will never become hot enough to initiate nuclear fusion. [20]

[The pressure due to a degenerate gas of non-relativistic fermions is

$$P = \frac{h^2}{20m}\left(\frac{3}{\pi}\right)^{2/3} n^{5/3}\,,$$

where m and n are the fermion mass and number density respectively.]

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2. A binary system is identified in which the brighter, main-seque primary star has a mass of $2\,M_{\odot}$, a luminosity of $15\,L_{\odot}$ and an effective temperature of 8000 K. Spectroscopy of the white dwarf companion star reveals that it has a similar temperature to the primary star, but its luminosity is only $3 \times 10^{-4}\,L_{\odot}$. The spectrum of the white dwarf appears redshifted compared to its companion by an amount equivalent to an excess recession velocity of $55\,\mathrm{km\,s^{-1}}$, that is not connected with its motion in the binary system.

   (a) Calculate the radius and mass of the white dwarf star. [20]

   (b) Assuming a likely composition, estimate how hot the interior of the white dwarf star would have to be to avoid being electron degenerate. Explain why such high temperatures are improbable. [25]

   (c) Using a suitable criterion, assess whether the degenerate electrons in the white dwarf star are relativistic. Explain whether your answer is likely to apply to all parts of the star. [15]

   (d) With reasons, estimate the mass of the main sequence progenitor star of the white dwarf and explain any difference between your answer and the mass calculated in part (a). [15]

   (e) If the white dwarf star were able to continuously accrete matter from the primary star, state what would happen to its radius and density and describe the nature of two physical mechanisms that might eventually trigger its collapse. [25]

   [You may assume that the Fermi momentum is given by $p_F = (3h^3 n/8\pi)^{1/3}$, where n is the fermion number density.]

3. (a) State the main source of energy that is radiated by a coo white dwarf star over its lifetime. Justify why you can largely neglect other sources of thermal energy that might be important in a normal (non-degenerate) star. [30]

(b) The luminosity of a cooling white dwarf star is given by $L = BMT_{\rm int}^{7/2}$, where $T_{\rm int}$ is the internal temperature, M is the mass of the star and $B = 10^{-30}\,{\rm W\,kg^{-1}\,K^{-7/2}}$ is a constant. Using *and stating* the Mestel treatment approximations, show that the time taken for a *carbon* white dwarf to cool to a *very* low luminosity $L_{\rm low}$ is given by

$$\tau = \frac{1}{20}\,\frac{k_B T_{\rm int}}{m_u}\,\frac{M}{L_{\rm low}}\,,$$

where m_u is an atomic mass unit. [40]

(c) A white dwarf star is observed *in the direction* of the young Hyades star cluster. The cluster has an age of 5×10^8 years. Assuming the white dwarf star is made of carbon and has a typical mass, then what is the smallest luminosity it can have if it is a genuine member of the cluster. [20]

(d) How might crystallisation of the white dwarf interior affect your answers to parts (b) and (c)? [10]

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4. (a) Explain what is meant by relativistic degeneracy. [

(b) Using the virial theorem, Newtonian gravity and explaining any other assumptions you make, show that there is a limiting mass, that can be supported by a gas of ideal, completely degenerate neutrons and calculate its value. [35]

(c) i. Sketch a diagram of stellar mass versus the logarithm of the central density ρ_c, for neutron stars with $10^{14} < \rho_c < 10^{19}\,\mathrm{kg\,m^{-3}}$. Label the axes with numerical values. Draw two curves that represent neutron stars that are governed by either “hard” or “soft” equations of state at high densities. Indicate any regions of instability. [20]

ii. Define what is meant by “hard” or “soft” equations of state. Describe possible examples of the physical mechanisms or compositions at the core of a neutron star that could lead to either a “hard” or “soft” equation of state. [25]

iii. Explain how finding accurate masses for neutron stars could rule out some possibilities for the equation of state at high density. [10]

[The pressure due to a degenerate gas of relativistic fermions is

$$P = \frac{hc}{8}\left(\frac{3}{\pi}\right)^{1/3} n^{4/3}\,,$$

where n is the fermion number density.]

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5. The energy density in the crust region of a neutron star can expressed as

$$u = n_N M(A, Z) + u_e + u_n$$

(a) Define the symbols in this equation and state the condition that can be used to establish the composition of the gas at equilibrium. [15]

(b) By considering the equilibrium between nuclei that have the same atomic number, but differ by 1 in their mass number, show that free neutrons emerge in the gas when

$$\frac{\partial M}{\partial A} = m_n c^2 \, ,$$

where m_n is the neutron mass. [25]

(c) The density at which the free neutrons emerge is about $3 \times 10^{14}\,\mathrm{kg\,m^{-3}}$, when $A = 122$ and $Z = 39$.

i. Calculate the Fermi energy of the electrons at this density and show that they are highly relativistic. [15]

ii. Explain in physical terms how and why the density dependence of the equation of state changes at densities (A) below where neutrons appear, (B) where they first appear and then (C) where their numbers increase significantly. [25]

iii. Explain why free protons cannot be produced in the gas until the density is much higher than $3 \times 10^{14}\,\mathrm{kg\,m^{-3}}$. [20]

[You may assume that the Fermi momentum is given by $p_F = (3h^3 n/8\pi)^{1/3}$, where n is the fermion number density.]