The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2012/13

Level II

Thursday 23$^{\text{rd}}$ May 2013, 09:30–11:30

PHYSICS/ASTROPHYSICS

PHY-20026

STATISTICAL MECHANICS AND SOLID STATE PHYSICS

Candidates should attempt ALL of PART A and TWO questions from PART B.

PART A yields 40% of the marks, PART B yields 60%.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

PART A Answer all TEN questions

A1 Define (a) lattice, (b) unit cell, (c) basis. [1,2,1]

A2 A vector

$$\mathbf{R} = \frac{A}{h}\mathbf{a} + \frac{B}{k}\mathbf{b} + \frac{C}{l}\mathbf{c},$$

where A, B, and C are constants, lies in the hkl plane if $A+B+C = 0$. Verify that $\mathbf{G}_{hkl} = h\mathbf{a}^* + k\mathbf{b}^* + l\mathbf{c}^*$ is perpendicular to the hkl plane. [4]

A3 Assuming that the free electrons in a metal constitute a classical Maxwellian gas, obtain an expression for the electronic contribution to the molar specific heat of a metal. [4]

A4 Write down – *in vector form* – (a) Ohm's Law and (b) an expression for the current density in terms of the number density and velocity of carriers. Define the terms you use. [4]

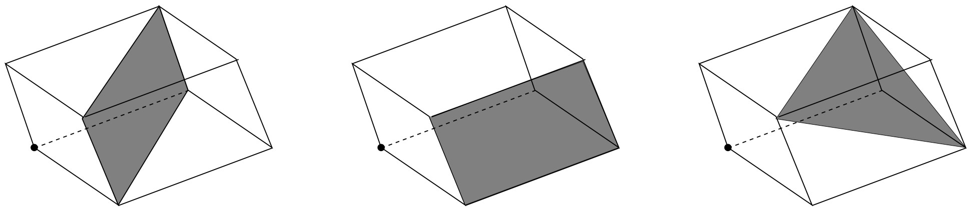
A5 Explain what is meant by a p-type semiconductor. [4]

A6 Calculate the number of conduction electrons per unit volume in lithium, assuming it has density 535 kg m^{-3} and valency 1. [4]

A7 A (very small) solid consists of a cube, within which the atoms form a cubic structure. Along each side of the solid there are 5 atoms. How many ways are there of placing 10 Schottky defects in the solid? [4]

A8 Sketch the Fermi-Dirac distribution for a gas at temperatur
0 K and (b) $T > 0$ K. Indicate the Fermi energy in your sketc

A9 Determine the Miller indices for the planes shown: [1,1,2]



A10 Describe, without using mathematical detail, the essential features of the Debye theory of specific heats. [4]

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PART B Answer TWO out of FOUR questions

B1 The Einstein theory of specific heats gives the following equation for the molar specific heat of a solid at temperature T:

$$C = 3R\left(\frac{\Theta_{\mathrm{E}}}{T}\right)^2 \frac{\exp[\Theta_{\mathrm{E}}/T]}{(\exp[\Theta_{\mathrm{E}}/T]-1)^2}$$

The Einstein temperature Θ_{E} is given by $\hbar\omega/k_{\mathrm{B}}$.

(a) i. What does ω represent? [5]

ii. What is meant by *high temperature* in the context of the Einstein theory? [5]

(b) Show that, in the high temperature limit, C approaches the classical value, $C = 3R$. [10]

(c) The Einstein temperature for lead is 67 K, whereas the Einstein temperature of carbon is 1450 K. The molar specific heats of these materials are measured at room temperature. *Without making a detailed numerical calculation*, which will have a C value close to $3R$? Explain your answer. [5]

(d) Determine the molar specific heat of carbon at temperature 100 K. [5]

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B2 (a) Explain why X-ray diffraction is more effective at locati… atoms rather than light atoms in a crystal.

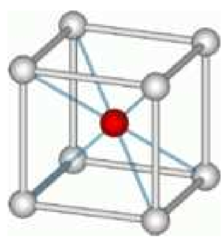
(b) The atomic scattering factor (also called the form factor) is given by

$$f = \sum_{n=1}^{Z} \int_0^\infty 4\pi r^2 \psi_n^2(r) \left\{ \frac{\sin[4\pi r(\sin\theta)/\lambda]}{[4\pi r(\sin\theta)/\lambda]} \right\} dr \ ,$$

where θ is the scattering angle, λ is the wavelength of the X-ray, Z is the atomic number of the scattering atom, $\psi_n(r)$ is the radial wave-function of the n th electron, properly normalised, and the sum is over all electrons in the atom. Show that, in the limit of small angle scattering and long wavelengths, $f = Z$. [10]

(c) Sketch the dependence of f on $[\sin\theta/\lambda]$. [5]

(d) i. The figure shows schematically the crystal structure of caesium chloride. Identify the lattice and the basis. [5]



ii. Discuss how the presence of the chlorine ions affects the nature of the X-ray diffraction pattern observed. [5]

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B3 (a) Write down an expression for (a) the internal energy U the partition function Z for the case that the energy lev non-degenerate. [

(b) Hence show that the internal energy U of a kg-mole of material may be expressed in the form:

$$U = RT^2 \, \frac{\partial \ln Z}{\partial T} \, ,$$

where R is the gas constant and T is temperature. [10]

(c) The partition function for a certain system may be expressed as

$$Z = AT^2 \, e^{\alpha T} \, ,$$

where A and α are constants.

i. Show that the internal energy per kg-mole is

$$U = RT \left(2 + \alpha T\right) \, . \qquad [10]$$

ii. Hence deduce an expression for the molar specific heat at constant volume. [5]

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B4 (a) Explain what is meant by the *Hall effect.*

(b) Assuming that the Lorentz force is

$$\mathbf{F} = Q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

in the usual notation, show that

i. the Hall field is proportional to the current and the applied field [10]

ii. and that, if electrons provide the current, the Hall coefficient is given by

$$R_{\mathrm{H}} = -\frac{1}{|e|\,n}$$

where n is the number of conduction electrons per m^{-3}. [5]

(c) A length of magnesium wire is used as a Hall probe to measure the magnetic field in a laboratory experiment. A voltage of 10 V is applied across the ends of the wire, which is 1 m long; the wire is inserted at right angles to the laboratory field and a Hall field of $3\,\mathrm{mV\ m^{-1}}$ is measured. Determine the magnitude of the magnetic field. [10]

The Hall coefficient for magnesium is $-0.83 \times 10^{-10}\mathrm{m^3\,C^{-1}}$ and its conductivity is $1.67 \times 10^{7}\ \Omega^{-1}\ \mathrm{m}^{-1}$.