# The Handbook of Mathematics, Physics and Astronomy Data is provided 

KEELE UNIVERSITY

EXAMINATIONS, 2012/13
Level II

Thursday $16^{\text {th }}$ May 2013, 09:30-11:30
PHYSICS

PHY-20009

NUCLEAR AND PARTICLE PHYSICS

Candidates should attempt to answer FOUR questions.

1. (a) State the charge numbers, strangeness, $S$, charm, $C$, and relative masses of the 6 quarks.
(b) Deduce, with justification, the quark structure of the following mesons, none of which contain a top quark:

$$
\begin{align*}
& \text { i. } \pi^{-} ; S=0, C=0, \text { mass }=139.6 \mathrm{MeV} / \mathrm{c}^{2},  \tag{10}\\
& \text { ii. } K^{+} ; S=1, C=0, \text { mass }=493.7 \mathrm{MeV} / \mathrm{c}^{2},  \tag{10}\\
& \text { iii. } D^{-} ; S=0, C=-1, \text { mass }=1869.3 \mathrm{MeV} / \mathrm{c}^{2},  \tag{10}\\
& \text { iv. } D_{S}^{+} ; S=1, C=1, \text { mass }=1869.3 \mathrm{MeV} / \mathrm{c}^{2},  \tag{10}\\
& \text { v. } B^{-} ; S=0, C=0, \text { mass }=5278.9 \mathrm{MeV} / \mathrm{c}^{2}, \tag{10}
\end{align*}
$$

(c) The $D^{-}$meson can be produced via the strong interaction:

$$
\pi^{-}+p \rightarrow D^{-}+\Lambda_{C}^{+}
$$

i. Discuss briefly the conservation of baryon number, $B$, strangeness and charm in this reaction and deduce the values of these quantities for the $\Lambda_{C}^{+}$particle.
ii. Assuming that no top or bottom quarks are involved in the reaction deduce, with explanation, the quark structure of the $\Lambda_{C}^{+}$particle. Write down the equation at the quark level and check that it follows the conservation laws.
2. (a) State what is meant by a magic number in the shell model comment on the relevance of magic numbers to isotopic stability.
(b) Why do the most stable light nuclei tend to have equal number of protons and neutrons?
(c) The lowest shell model states are, in order of increasing energy:

$$
\left.\begin{array}{c}
1 s_{1 / 2} \quad 1 p_{3 / 2}
\end{array} 1 p_{1 / 2} \quad 1 d_{5 / 2} \quad 2 s_{1 / 2} \quad 1 d_{3 / 2}\right)
$$

i. Draw a shell model diagram for ${ }^{41} \mathrm{Ca}$ and predict with explanation its ground state spin and parity.
ii. Using the shell diagram you have drawn, predict the spin and parity of the first 2 excited states of ${ }^{41} \mathrm{Ca}$. Explain your answer.
(d) How do the following properties of ${ }^{41} \mathrm{Sc}$ compare with those of ${ }^{41} \mathrm{Ca}$ ? Justify your answer.
i. Ground state spin and parity.
ii. Stability.
3. The energies in units of keV , spins and parities of low lying exc states in some nuclei are given as follows:

| State / Nuclei | ${ }^{108} \mathrm{Pd}$ | ${ }^{138} \mathrm{Ba}$ |  | ${ }^{208} \mathrm{~Pb}$ |  | Pu |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1^{\text {st }}$ | 433 | $2^{+}$ | 1435 | $2^{+}$ | 2614 | $3^{-}$ | 44 | $2^{+}$ |
| $2^{\text {nd }}$ | 931 | $2^{+}$ | 1898 | $4^{+}$ | 3197 | $5^{-}$ | 147 | $4^{+}$ |
| $3^{\text {rd }}$ | 1048 | $4^{+}$ | 2090 | $6^{+}$ | 3475 | $4^{-}$ | 306 | $6^{+}$ |
| $4^{\text {th }}$ | 1052 | $0^{+}$ | 2189 | $1^{+}$ | 3708 | $5^{-}$ | 518 | $8^{+}$ |
| $5^{\text {th }}$ | 1314 | $0^{+}$ | 2203 | $6^{+}$ | 3919 | $6^{-}$ | 778 | $10^{+}$ |
| $6^{\text {th }}$ | 1335 | $3^{+}$ | 2217 | $2^{+}$ | 3946 | $4^{-}$ | 832 | $3^{-}$ |

(a) State and explain the spin and parity for the ground state of these nuclei.
(b) i. Using the information in the table above, state and explain for which nucleus the excited states correspond best to a sequence of rotational states. Explain why the others do not fit as well.
ii. Calculate, using the first excited state, an estimate for the moment of inertia of the rotational nucleus of part (b)i. [15] (Hint: the expectation value of the square of the total nuclear spin, $\left\langle\vec{I}^{2}\right\rangle=\hbar^{2} I(I+1)$.)
iii. Which state(s) in this rotational nucleus is (are) not explained by rotational states?
(c) i. Using the information in the table above, state and explain for which nucleus the excited states correspond best to a sequence of vibrational states.
ii. Calculate, using an appropriate excited state, an estimate for the phonon frequency for the vibrational nucleus of part (c)i.
/Cont'd
4. (a) Write down equations for the $\alpha$ and $\beta^{-}$decays of ${ }^{223} \mathrm{Fr}$.
(b) The decay constants for the $\alpha$ and $\beta^{-}$decays are $3.15 \times 10^{-8}$ and $5.25 \times 10^{-4} \mathrm{~s}^{-1}$, respectively. Calculate the total
i. decay constant and
ii. half-life
of ${ }^{223} \mathrm{Fr}$.
(c) A pure sample of ${ }^{223} \mathrm{Fr}$ contains $10^{10}$ nuclei. Calculate the number of
i. $\alpha$ particles and
ii. $\beta$ particles
emitted from the decay of ${ }^{223} \mathrm{Fr}$ in a period of 2 minutes, stating any approximations made.
(d) ${ }^{223} \mathrm{Fr}$ has a $3 / 2^{-}$ground state. $\beta$ decays of this nucleus populate the $3 / 2^{-}$and $5 / 2^{+}$excited states in the daughter nucleus.
i. Determine for both daughter nucleus states, whether the decay to the state is allowed or its degree of forbiddenness.
ii. Hence state and explain which daughter state ${ }^{223} \mathrm{Fr}$ will predominantly decay to.
5. (a) For the following reactions, replace the question mark with appropriate nucleus or particle and name which reaction category they belong to.

$$
\text { i. }{ }^{27} \mathrm{Al}(\mathrm{~d}, ?){ }^{25} \mathrm{Mg}
$$

ii. ${ }^{20} \mathrm{Ne}(?, \alpha)^{20} \mathrm{Ne}$
iii. ${ }^{24} \operatorname{Mg}(\alpha, \gamma)$ ?
(b) In a particular reaction, a projectile of mass, $m_{P}$, and kinetic energy, $T_{P}$, is incident on a stationary target nucleus. Following the reaction, a single ejectile of mass, $m_{E}$ and kinetic energy, $T_{E}$ is emitted at $90^{\circ}$ to the projectile direction, leaving a recoiling residual nucleus of mass, $m_{R}$ and kinetic energy, $T_{R}$. Assuming that the motion is non-relativistic, show that:

$$
\begin{equation*}
m_{R} T_{R}=m_{P} T_{P}+m_{E} T_{E} \tag{30}
\end{equation*}
$$

(c) Hence show that the $Q$ value for this reaction:

$$
\begin{equation*}
Q=T_{E}\left(1+\frac{m_{E}}{m_{R}}\right)-T_{P}\left(1-\frac{m_{P}}{m_{R}}\right) \tag{20}
\end{equation*}
$$

(d) If the result of part (c) is applied to elastic scattering and the mass of the target nucleus is three times the mass of the projectile, determine the fraction of the projectile kinetic energy, which is lost in the scattering process.
6. (a) Describe the process of neutron-induced fission, explaining two stages by which it occurs.
(b) A plot of number of fragments versus mass number, for thermal neutron induced fission on ${ }^{239} \mathrm{Pu}$ shows a broad peak around mass number 138. Assume that on average each fission produces one more neutron than it uses. Describe and explain another broad peak in the plot.
(c) Show that the ratio of the two fission fragment kinetic energies is approximately inversely proportional to the ratio of their masses, stating any approximations made.
(d) Write down the reaction for the neutron induced fission of ${ }^{239} \mathrm{Pu}$ into ${ }^{138} \mathrm{Xe}$ assuming that two neutrons are released during the fission.
(e) Using the information below, calculate the Q value for this fission reaction.
(f) Calculate the kinetic energy of the ${ }^{138} \mathrm{Xe}$ fragment.

Additional data:
Neutron and atomic masses in $u\left(1 u=931.5 \mathrm{MeV} / \mathrm{c}^{2}\right)$

$$
\begin{array}{rr}
\mathrm{n} & 1.008665 \\
{ }^{100} \mathrm{Zr} & 99.917762 \\
{ }^{101} \mathrm{Zr} & 100.921140 \\
{ }^{102} \mathrm{Zr} & 101.922981 \\
{ }^{138} \mathrm{Xe} & 137.913989 \\
{ }^{239} \mathrm{Pu} & 239.052158
\end{array}
$$

