The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2012/13

Level I

Thursday 17th January 2013, 16.00-18.00

PHYSICS/ASTROPHYSICS

PHY-10024

NATURE OF MATTER

Candidates should attempt ALL of PART A and TWO questions from PART B.

PART A yields 40% of the marks, PART B yields 60%.

NOT TO BE REMOVED FROM THE EXAMINATION HALL

PART A Answer all TEN questions

A1 How many H_2O molecules are there in a 0.5-litre water bottle? [4]

A2 Helium gas, initially at atmospheric pressure ($P = 1.01 \times 10^5\,\mathrm{Pa}$), is compressed adiabatically to one quarter of its initial volume. Determine the pressure after the compression. [4]

A3 The van der Waals equation of state is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \ .$$

Sketch the phase diagram (i.e. P against V) for a 'real' (as opposed to 'ideal') gas that obeys the van der Waals equation of state; your sketch should include a $P - V$ curve for temperatures well below, and well above, the critical temperature T_c. [4]

A4 A gas at pressure P is contained in a cylindrical vessel. The gas does work on a friction-free piston by raising it by a small distance dx. Show that the work done by the gas is $dW = P\,dV$, where dV is the change in gas volume. [4]

A5 The potential energy, PE, of a particle due to the van der Waals bond (referred to as the Lennard-Jones potential) is described by the formula:

$$PE = -\frac{A}{r^6} + \frac{B}{r^{12}} \ ,$$

where A and B are constants and r is the distance between the particles. Sketch the variation of PE with r; include on your diagram the variation of each of the terms contributing to the total potential energy; indicate on your diagram the equilibrium separation between particles. [4]

A6 X-rays of wavelength 0.158 nm are reflected from a cubic CsCl crystal; the first order reflection occurs at 15.7°. What value does this give for the inter-planar spacing of CsCl? [4]

A7 Photons of wavelength 590 nm are emitted by a 50 W sodium lamp. How many photons are emitted per second? [4]

A8 It is desired to study an object, of dimensions 10^{-14} m, in a neutron diffraction experiment. What is the minimum velocity that neutrons must have in this experiment? [4]

A9 A certain element having atomic number Z is bombarded with high energy electrons and monochromatic X-rays having wavelength of 0.21 nm are emitted. Identify the element. [Use Moseley's law: $\lambda = \frac{4}{3R}\frac{1}{(Z-1)^2}$ where $R = 1.09737 \times 10^7\,\mathrm{m}^{-1}$.] [4]

A10 The mass of a ^{1}H atom is 1.007825 atomic mass units (amu), the mass of a neutron is 1.008665 amu, while the mass of a ^{22}Ne atom is 21.991383 amu. If 1 amu $= 1.6604 \times 10^{-27}$ kg, calculate the binding energy of a ^{22}Ne atom in MeV. [4]

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PART B Answer TWO out of FOUR questions

B1 (a) In the context of kinetic theory, what is a *degree of freedom*? [2]

(b) State the theorem of Equipartition of Energy. [3]

(c) Show that, for a simple 3-dimensional crystalline solid, the Equipartition Theorem gives the result $C_{\mathrm{v}} = 3R$ for the molar specific heat. [6]

(d) Sketch the temperature-dependence of the specific heat at constant volume, C_{v}, for a solid. Explain the basic features of the plot. [6]

(e) The specific heat at constant volume of an unknown solid is measured at high temperature to be $319.5\,\mathrm{J\,kg^{-1}\,K^{-1}}$. Estimate its molecular weight. [8]

(f) If the solid in part (c) were 2-dimensional rather than 3-dimensional, what would the specific heat be? [5]

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B2 (a) Describe briefly what is meant by (i) an ionic bond, (ii) a co-
lent bond, (iii) a van der Waals bond, (iv) co-ordination number. [4 × 2]

(b) For ionic crystals, the total potential energy per ion may be expressed as

$$PE = -\alpha \frac{e^2}{4\pi\epsilon_0 r} + \frac{B}{r^{12}} \,,$$

where α and B are constants.

i. Explain briefly the origin of the two terms on the right hand-side. [4]

ii. There is a minimum in the potential energy at the inter-molecule spacing r_0. Derive an expression for B in terms of r_0. [4]

(c) Which of the bonds listed in part (a) are important in liquid nitrogen, which consists of nitrogen molecules N_2 ? [4]

(d) The latent heat of vaporisation of liquid nitrogen is $201\,\mathrm{kJ\,kg^{-1}}$. Estimate the strength of the bond in liquid nitrogen, and hence deduce the nature of the bond in terms of the above three alternatives. [10]

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B3 (a) State the three postulates of Bohr's model of the H atom.

(b) Bohr's model can explain Rydberg's formula for the wavelength of hydrogen spectral lines:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where $R = 1.09737 \times 10^7\,\mathrm{m}^{-1}$. What do n_1 and n_2 represent in Bohr's model? [2]

(c) A laser uses the transition from $n_2 = 2$ to $n_1 = 1$ to produce its light. What is the wavelength of the light? [2]

(d) Outline Einstein's theory of the photo-electric effect. [10]

(e) Can the laser described in part (c) be used in a photo-electric experiment with a metal, whose work function is 5.0 eV? Explain your answer. [5]

(f) If the laser is suitable, what is the maximum kinetic energy of the electrons ejected? If not, what is the maximum wavelength that will produce the photo-electric effect? [5]

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B4 (a) What condition on the masses must be fulfilled for a parent nucleus X to undergo an $\alpha-$decay and produce a daughter nucleus Y? [4]

(b) The number of parent nuclei at time t is given by

$$N = N_0 e^{-\lambda t} ,$$

where N_0 is the initial number of parent nuclei and λ is the decay constant. The time required for the number of parent nuclei to drop to 50% of the initial number is called the half-life, $t_{1/2}$. Show that the half-life and decay constant are related by the following equation:

$$\lambda = \frac{\ln 2}{t_{1/2}}.$$

[6]

(c) The half-life for the $\alpha-$decays of $^{238}_{92}\text{U}$ and $^{226}_{88}\text{Ra}$ are 4.5×10^9 years and 1.62×10^3 years, respectively. If there are 10^9 atoms of each $^{238}_{92}\text{U}$ and $^{226}_{88}\text{Ra}$ initially, how many atoms of each isotope are left after 1000 years? [10]

(d) What are the daughter nuclei produced in these decays? [2×2]

(e) Discuss a potential application of these decays and its limitations? [6]