EXAMINATION PAPER CONTAINS STUDENT'S ANSWERS

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The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2012/13

Level I

Monday 14th January 2013, 09:30–11:30

PHYSICS/ASTROPHYSICS

PHY-10022

MECHANICS, GRAVITY and RELATIVITY

Candidates should attempt ALL of PARTS A and B, and TWO questions from PART C. PARTS A and B should be answered on the exam paper; PART C should be answered in the examination booklet which should be attached to the exam paper at the end of the exam with a treasury tag. PART A yields 16% of the marks, PART B yields 24%, PART C yields 60%.

Please do not write in the box below

A		C1		Total
B		C2		
		C3		
		C4		

NOT TO BE REMOVED FROM THE EXAMINATION HALL

PART A Tick one box by the answer you judge to be

A1 The units of energy are equivalent to:

☐ $m^2\ s^{-2}$ ☐ $kg\ m^2\ s^{-2}$ ☐ kg m s ☐ $kg\ m^2\ s$

A2 Work is calculated from:

☐ dE/dt ☐ $\int F\,dt$ ☐ $P\,dm$ ☐ $\int F\,dx$ [1]

A3 A flywheel undergoes 5 revolutions per second. Its angular velocity is (in $\text{rad}\,\text{s}^{-1}$):

☐ 5π ☐ 2.5π ☐ 31.4 ☐ 3.14 [1]

A4 A canal boat is pulled by a horse; the tension in the rope is 800 N and the rope makes an angle of 20° to the direction of travel. To keep the boat straight, the rudder must exert a lateral force of:

☐ 752 N ☐ 153 N ☐ 851 N ☐ 274 N [1]

A5 A particle moving with distance $x = 2t^2$ has a velocity of:

☐ $8t$ ☐ $4t^2$ ☐ $4t$ ☐ 4 [1]

A6 The dot product of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is equivalent to:

☐ $ab\cos\theta$ ☐ $\boldsymbol{a} \wedge \boldsymbol{b}$ ☐ $ab\sin\theta\,\hat{\boldsymbol{n}}$ ☐ $\sqrt{a^2+b^2}$ [1]

A7 A 1-kg hammer strikes a 10-g nail exerting a force of 80 N. The nail exerts a force on the hammer of:

☐ 0.8 N ☐ 8 N ☐ 80 N ☐ 100 N [1]

A8 The angular momentum vector points:

☐ along the axis of rotation
☐ perpendicular to the torque
☐ in the plane of the body
☐ in the direction of θ [1]

/Cont'd

A9 If the angle turned by a wheel is described by $\theta = 30t$ … lar velocity is:

- ☐ increasing ☐ decreasing ☐ zero ☐ constant

A10 Angular momentum is given by:

- ☐ rm^2 ☐ $I\omega$ ☐ $\int r^2 \, dm$ ☐ $d\tau/d\theta$ [1]

A11 Which of the following quantities is not necessarily conserved?:

- ☐ energy ☐ momentum
- ☐ angular momentum ☐ force [1]

A12 The expression for gravitational potential energy mgh is valid:

- ☐ always ☐ for small m ☐ for small h ☐ never [1]

A13 Kepler's laws state that planetary orbits:

- ☐ are always circular ☐ have a fixed speed
- ☐ don't depend on mass ☐ are elliptical [1]

A14 Special Relativity applies only to:

- ☐ heavy bodies ☐ uniform relative motions
- ☐ accelerating bodies ☐ velocities near c [1]

A15 Which of the following has the greatest mass?

- ☐ An infra-red photon ☐ An optical photon
- ☐ A UV photon ☐ An X-ray photon [1]

A16 A light beam travels at c as seen by one observer. As seen by a rocket, travelling at $0.5\,c$ with respect to the first observer, the light beam appears to travel at:

- ☐ $0.5\,c$ ☐ $c/\sqrt{0.75}$ ☐ $c/(1+0.25)$ ☐ c [1]

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PART B Answer all EIGHT questions

B1 A car travels at a constant speed, then brakes hard as the driver sees a rabbit, then gradually accelerates again to the previous speed, and continues. Sketch the acceleration, speed and distance travelled as functions of time. [3]

B2 A crane lifts 500 kg vertically at a rate of 2 $\mathrm{m\,s^{-1}}$. What is the power exerted by the crane's motor? [Take $g = 9.8\ \mathrm{m\,s^{-2}}$.] [3]

B3 A 0.3-kg mass is swung in a circle by a 2-m piece of string at one revolution per second. What is its angular momentum? [3]

B4 Define the quantity torque. [3]

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B5 A boy pushes a 50-kg cart up an incline, 30° to the hor
What is the minimum force with which the boy needs to pu
[Take $g = 9.8\ \mathrm{m\,s^{-2}}$.] [3]

B6 What is the gravitational potential 1000 km away from a mass of 10^{22} kg? [3]

B7 An electron moves so that its total energy is twice its rest-mass energy. What is its speed? [3]

B8 What is the momentum of a proton travelling at $0.8c$? [3]

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PART C Answer TWO out of FOUR questions

C1 A particle has a momentum as a function of time given by $p = 2t$ kg m s^{-1}. What is the force acting on the particle? [4]

The particle as a mass of 0.1 kg. Assuming that it starts at $x = 0$ at $t = 0$, find the distance, x, as a function of time. [6]

Sketch a plot of the power being gained by the particle as a function of time. [6]

What is the power being gained by the particle at the time $t = 2$ s? [2]

What is the energy transfered to the particle over the first two seconds? [6]

If the particle's mass was not constant, but was decreasing with time, with everything else above remaining the same, would the energy transfered over the first two seconds be higher or lower? Justify your answer. [6]

C2 A medieval catapult is made of a thin wooden beam of length L and mass M rotating about an axis one third of the way from one end. Starting from $I = \int r^2 \, dm$ show the moment of inertia of the beam is $I = ML^2/9$. [12]

A counterweight mass of 100 kg is attached to the shorter end of the beam. The beam is 6 m long and has a mass of 30 kg. What is the total moment of inertia? [8]

The catapult starts from rest. A torque of 1300 N m is applied for 2 s, causing a light projectile to be thrown from the longer arm of the catapult. What is the linear velocity of the projectile as ejected at 2 s? [10]

/Cont'd

C3 Given that a particle in circular motion has a position giv vector

$$\boldsymbol{r} = A\cos(\omega t)\hat{\boldsymbol{\imath}} + A\sin(\omega t)\hat{\boldsymbol{\jmath}}$$

(for orthogonal directions $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ where A is a constant and t the time) show that the centripetal acceleration is $a = -r\omega^2$. [6]

Hence show that a space station in a circular orbit around a mass M at a distance r has an orbital speed given by $v = \sqrt{GM/r}$. What is this speed for a space station that is in a circular orbit 100 km above a planet that has a mass of 1.0×10^{24} kg, a radius of 4000 km, and no atmosphere? [6]

A projectile is fired backwards from the space station with a velocity of 200 m s^{-1} relative to the space station, in the opposite direction to the space station's orbital motion. Describe qualitatively the future path of the projectile. Also, describe the effect on the space station of launching the projectile. [6]

Such a projectile ends up hitting the planet. At what speed does it hit? [8]

Would you expect this speed to be greater than or less than that of a meteorite falling onto the planet from deep space? Justify your answer. [4]

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C4 Using the Lorentz transforms, show that a length measu[illegible] a frame S, appears to be contracted to a length l/γ when [illegible] from a frame S' moving with respect to S. [1[illegible]

Cosmic-ray muons are created 5 km above the Earth's surface and travel downwards at a speed of $0.995c$. These particles are unstable and decay with a typical lifetime of 2.2×10^{-6} s. How far does a muon typically travel, as seen from the muon's frame? As viewed from the muon's frame, does it get as far as the Earth's surface? Justify and explain your answers. [10]

Now consider the perspective of an observer stationary on Earth's surface, watching the muon's being created 5 km above. Would that observer agree that the muons would travel as far as Earth's surface? Justify and explain your answer. [6]

A detector stops 100 cosmic-ray muons. Given that the rest-mass of the muon is 1.88×10^{-28} kg, what is the energy absorbed by the detector? [4]