The Handbook of Mathematics, Physics and Astronomy Data is provided

KEELE UNIVERSITY

EXAMINATIONS, 2012/13

Level I

Thursday 23rd May, 9.30-11.30

PHYSICS/ASTROPHYSICS

PHY-10021

ELECTRICITY & MAGNETISM

Candidates should attempt ALL of PART A and ONE question from each of PARTS B and C.

PART A yields 40% of the marks, PART B yields 30%, PART C yields 30%

NOT TO BE REMOVED FROM THE EXAMINATION HALL

PART A Answer all TEN questions

A1 Calculate the ratio of the electric force to the gravitational force between an electron and a proton in a hydrogen atom. [4]

A2 What is the work done by an external force in order to bring a $+1\mu$C test charge from infinity to a distance of 10 cm from a point charge of $+50\mu$C? [4]

A3 Two 8μF capacitors are connected in parallel to a 12 V battery. What is the combined capacitance and how much work is done by the battery to charge both capacitors? [4]

A4 A 24 μC electric charge is placed at the centre of a cubic box. What is the electric flux through one side of the cubic box? [4]

A5 A charge of 0.5 C flows through a resistor of $R = 50\Omega$ in 5 seconds. How much energy is dissipated in the resistor? [4]

A6 A proton beam moves through a uniform magnetic field with magnitude $B = 2.0\,\mathrm{T}$, directed along the positive z-axis. The protons have a velocity of $3.0 \times 10^5(0.5\vec{i} + 0.866\vec{k})\,\mathrm{m\,s^{-1}}$ in the xz plane. Find the magnitude and direction of the force on a proton. [4]

A7 A closed loop encircles several electrical conductors. The line integral $\oint \vec{B} \cdot \vec{dl}$ around the loop is $1.2566 \times 10^{-4}\,\mathrm{T\,m}$. Calculate the net current in the conductors. [4]

A8 An ideal solenoid has 200 turns over a 0.1 m length and carries a current of 5 A. Calculate the B field inside the solenoid. [4]

A9 A loop of conducting wire, of area $2\,\mathrm{m}^2$ has a perpendicular magnetic field passing through it. What is the induced voltage as a function of time in the loop if the magnetic field varies as $B = (5t^2 + 30)$ Tesla (where t is the time)? [4]

A10 The current in a LR circuit decays as $I = I_0 \exp(\frac{-tR}{L})$. The resistance in the circuit is $R = 0.2\Omega$, and the current decays to half its initial value in 0.05 s. Calculate the inductance of the circuit. [4]

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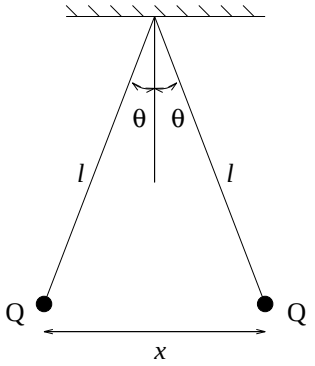
PART B Answer ONE out of TWO questions

B1 (a) Using either Coulomb's or Gauss's law derive an expression for the electric field $\vec{E}$ and the electric potential V at a distance r from a point charge Q. [6]

(b) Two identical conducting balls of mass M are hung from silk threads of length l and carry identical charges Q as shown in figure below. Assume that θ is so small that $\tan\theta \approx \sin\theta$.



i. Sketch a diagram showing the forces acting on either of the balls. [6]

ii. Show that,

$$x = \left[\frac{Q^2 l}{2\pi\epsilon_o M g}\right]^{\frac{1}{3}}$$

where x is the equilibrium separation between the balls. [12]

iii. If the threads are of 1 m length and hang at an angle of 5 degrees to the vertical, while the mass of each ball is 0.1 kg, calculate the charge on each ball. [6]

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B2 (a) State and explain Gauss's law concerning electric fields a charges. [6]

(b) A spherically symmetric distribution of charge has a charge density ρ given as follows:

$$\rho = \rho_o \qquad r \leq R,$$

$$\rho = 0 \qquad r > R,$$

where $\rho_o = \dfrac{3Q}{4\pi R^3}$ is a constant.

i. What is the total charge contained within the charge distribution? [3]

ii. Derive an expression for the electric field where $r \leq R$. [12]

iii. Show that, for the region defined by $r > R$, the electric field is identical to that produced by a point charge Q. [3]

iv. Sketch how the magnitude of the electric field varies with radius. [6]

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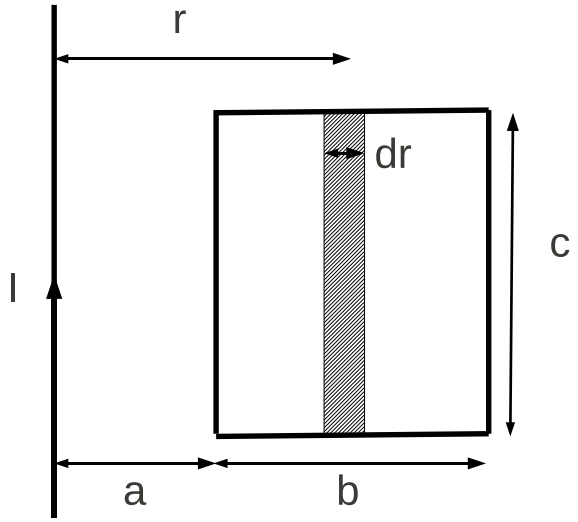
PART C Answer ONE out of TWO questions

C1 (a) State and explain the Faraday and Lenz laws of electromagnetic induction. [9]

(b) A current I passes through an infinite wire, at a distance a from a wire loop of width b and height c, as shown below.



i. What is the magnitude of the field $\vec{B}$ at a distance r from the wire? (Express your answer in terms of the variables given in the figure.) [3]

ii. State the direction of the $\vec{B}$ field at the position of the shaded strip, given the direction of the current shown above. [3]

iii. What is the magnetic flux $d\phi$ through the shaded strip of width dr and height c? [3]

iv. Evaluate the total flux through the loop. [6]

v. If the current varies with time as $I = I_0 \cos(\omega t)$, derive an expression for the induced EMF in the loop versus time. [6]

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C2 (a) For an LC circuit, plot how the charge on a capacitor and the current through the circuit vary with time. [6]

(b) A 25μF capacitor is initially charged by connecting it to a 50 V DC power supply. Once fully charged (at $t = 0$) it is then connected to a 10 mH inductor in a closed LC circuit, whereby the charge on the capacitor varies as $Q = Q_0 \cos(\omega t)$.

i. What is the initial charge on the capacitor, Q_0? [3]

ii. Calculate the angular frequency of oscillation in the LC circuit. [3]

iii. Calculate the charge on the capacitor and the current in the circuit at $t = 1.2$ ms. [6]

iv. What is the total energy stored in the LC circuit at any time? At $t = 1.2$ ms how much energy is stored in the inductor and how much in the capacitor? [6]

v. If a source of resistance is added to the circuit, state how the total energy stored in the circuit would vary with time. How does the angular frequency change if the resistance is increased? [6]